Fiscal Policy and Labor Mobilization:
Labor Demand and Supply Effects

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November 2025

Preliminary: Results subject to change

Government Spending (G) Shocks: Labor Demand or Supply?

Conventional/Keynesian wisdom:

- Government spending $\rightarrow$ more jobs $=$ labor **demand**

Neo-classical and New Keynesian models:

- G $\rightarrow$ negative wealth effect $\rightarrow$ labor **supply**
- Some liquidity effects in TANK/HANK models
- Labor demand second order

Empirical challenge:

- Employment shifts include both demand and supply
- Wages are informative but not sufficient to separate demand from supply

Our Approach

Theory: OE-NK model, firms with decreasing returns:

- **Sequence-space** labor demand and supply curves

Labor demand:

- Cross sectional data helps isolate labor demand responses to G shocks

Labor supply:

- Calibrated **sufficient statistic** for labor supply response

Application:

- **New data** on WWII county employment and wages; bond purchases, vacancies, labor mobility
- New **identification** of shifts in G across labor markets

Related Literature and Contribution

Theory: Fiscal policy in a currency union:

Representative references: Galí and Monacelli (2005); Farhi and Werning (2012, 2019); Nakamura and Steinsson (2014)

This paper: Focus on labor demand/supply; sufficient statistics theory $\leftrightarrow$ data

Cross-sectional multipliers:

Representative references: Chodorow-Reich et al (2012); Wilson (2012); Chodorow-Reich (2019); Brunet (2024)

This paper: Disentangle labor demand/supply; sufficient statistics approach; new shift-share instrument in WWII context

Labor demand vs. supply in response to G shocks:

Empirical: Perotti (2007, 2011, 2019) vs. Ramey (2011, 2016); Monacelli, Perotti, & Trigari (2010); Nekarda & Ramey (2011, 2020); Bills, Klenow & Malin (2013); Ramey and Zubairy (2018). Theory: Michaillat (2014); Ravn and Sterk (2017)

This paper: Disentangle labor demand/supply; panel vs. time series

1. Theory: Fiscal Policy in a Currency Union

2. New archival data: WWII labor markets

3. Empirical strategy

4. Estimating labor demand and supply

Households: Utility

Two regions $i \in \{A, B\}$ of equal size, each with a measure 1 of ex-ante identical households that can consume from either region but can only work within their own region.

Lifetime utility

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{it}^{1+\varphi}}{1+\varphi} \right) \right\}.$$

Consumption is a CES aggregator of consumption from two regions

$$C_{it} \equiv \left(\sum_{i' \in \{A,B\}} \omega_i(i')^{\frac{1}{\gamma}} C_{it}(i')^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}.$$

- If $\omega_i(i) > 0.5 \Rightarrow$ home bias.

Households: Optimality

$$P_{it}C_{it} + Q_{t,t+1}B_{i,t+1} = W_t(i)L_{it} + B_{it} + D_{it} - T_t(i) \qquad \text{(Budget)}$$

$$C_{it}(i') = \omega_i(i')\left(\frac{P_t(i')}{P_{it}}\right)^{-\gamma} C_{it}. \qquad \text{(Consumption basket)}$$

$$\chi\frac{L_{it}^{\varphi}}{C_{it}^{-\sigma}} = \frac{W_t(i)}{P_{it}}. \qquad \text{(Intratemporal optimality)}$$

$$Q_{t,t+k} = \beta^k \mathbb{E}_t\left\{\frac{P_{it}}{P_{i,t+k}}\left(\frac{C_{i,t+k}}{C_{it}}\right)^{-\sigma}\right\}. \qquad \text{(Euler)}$$

- $P_t(i')$ is price of goods from region i'.
- P_{it} is household CPI, defined by $P_{it} \equiv \left(\sum_{i'\in\{A,B\}}\omega_i(i')P_t(i')^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$.

Firms: Production

<u>Final good firm</u> sells final good to households and government after, produced from intermediate goods with CES production function

$$Y_t(i) = \left(\int_0^1 x_t(i,j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}.$$

Minimizing costs $\rightarrow$ demand for variety j:

$$x_t(i,j) = \left(\frac{p_t(i,j)}{P_t(i)} \right)^{-\eta} Y_t(i).$$

<u>Intermediate goods firms</u> hire labor in competitive markets of their own region, to produce with a **decreasing returns to scale** production function

$$y_t(i,j) = \ell_t(i,j)^{\alpha}.$$

Firms: Price Setting

Pricing decisions: Calvo price rigidities, with fraction $1-\theta$ adjusting. Adjusting firms set:

$$\max_{p_t(i,j)} \quad \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[p_t(i,j) y_{t+k|t}(i,j) - W_{t+k}(i) \ell_{t+k}(i,j) \right] \right\}$$

$$\text{s.t.} \quad y_{t+k}(i,j) = \ell_{t+k}(i,j)^{\alpha} \quad \text{and} \quad y_{t+k|t}(i,j) = \left(\frac{p_t(i,j)}{P_{t+k}(i)} \right)^{-\eta} Y_{t+k}(i).$$

Optimal price is

$$p_t(i,j) = \frac{\eta}{\eta-1} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \frac{\theta^k Q_{t,t+k} y_{t+k}(i,j)}{\mathbb{E}_t \left\{ \sum_{h=0}^{\infty} \theta^h Q_{t,t+h} y_{t+h}(i,j) \right\}} MC_{t+k}(i,j) \right\}.$$

Current pricing affects future marginal costs

Government and Monetary Authority

Government local purchases:

$$\log G_t(i) = \rho_g \log G_{t-1}(i) + (1-\rho_g) \log \bar{G}(i) + \varepsilon_t(i).$$

Government budget constraint:

$$\sum_{i\in\{A,B\}} P_t(i) G_t(i) = \sum_{i\in\{A,B\}} T_t(i) + Q_t B_{t+1} - B_t.$$

Monetary authority:

$$i_t = \rho_i \, i_{t-1} + (1-\rho_i)(\varrho + \phi_\pi \pi_t + \phi_y \hat{y}_t),$$

where $i_t \equiv -\log Q_{t,t+1}$, $\varrho \equiv -\log \beta$, $\pi_t \equiv \log P_t - \log P_{t-1}$, and $\hat{y}_t \equiv \log Y_t - \log \bar{Y}$.

Market Clearing

$$Y_t(i') = \sum_{i \in \{A,B\}} C_{it}(i') + G_t(i'), \quad \text{for } i' \in \{A, B\}.$$

$$y_t(i,j) = x_t(i,j), \quad \text{for } i \in \{A, B\} \text{ and } \forall j.$$

$$\int_0^1 \ell_t(i,j)dj = L_{it}, \quad \text{for } i \in \{A, B\}.$$

$$\sum_{i \in \{A,B\}} B_{it} = B_t.$$

Dynamic Labor Demand (for price adjusters)

Pricing equation, production function, consumer demand, combine to

Dynamic labor demand function

$$\ell_t(i,j) = A\,Y_t(i)^{\frac{1}{\alpha+\eta-\alpha\eta}} \left(\frac{P_t(i)}{P_{it}}\right)^{\frac{\eta}{\alpha+\eta-\alpha\eta}} \left(\mathbb{E}_t\left\{\sum_{m=0}^{\infty} f_{t+m}(\theta,Q,Y,P)\left(\frac{Y_{t+m}(i)}{Y_t(i)}\left(\frac{P_{t+m}(i)}{P_t(i)}\right)^{\eta}\right)^{\frac{1}{\alpha}} \frac{P_{i,t+m}}{P_{i,t}}\; w_{t+m}(i)\right\}\right)^{-\frac{\eta}{\alpha+\eta-\alpha\eta}}.$$

- $w_{t+m}(i) \equiv \frac{W_{t+m}(i)}{P_{i,t+m}}$; $A \equiv \left(\alpha\,\frac{\eta-1}{\eta}\right)^{\frac{\eta}{\alpha+\eta-\alpha\eta}}$; $f_{t+m}(.) \rightarrow$ discounting and weighting function of aggregates.

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Sequence-space labor demand function

- $w_{t+m}(i) \equiv \frac{W_{t+m}(i)}{P_{i,t+m}}$; $A \equiv \left(\alpha \frac{\eta-1}{\eta}\right)^{\frac{\eta}{\alpha+\eta-\alpha\eta}}$; $f_{t+m}(.) \rightarrow$ discounting and weighting function of aggregates.

Dynamic Labor Demand (for price adjusters)

Pricing equation, production function, consumer demand, combine to

Dynamic labor demand function

$$\ell_t(i,j) = A\, Y_t(i)^{\frac{1}{\alpha+\eta-\alpha\eta}} \left(\frac{P_t(i)}{P_{it}}\right)^{\frac{\eta}{\alpha+\eta-\alpha\eta}} \left(\mathbb{E}_t \left\{ \sum_{m=0}^{\infty} f_{t+m}(\theta, Q, Y, P) \left(\frac{Y_{t+m}(i)}{Y_t(i)} \left(\frac{P_{t+m}(i)}{P_t(i)}\right)^{\eta}\right)^{\frac{1}{\alpha}} \frac{P_{i,t+m}}{P_{i,t}} w_{t+m}(i) \right\}\right)^{-\frac{\eta}{\alpha+\eta-\alpha\eta}}.$$

Current and future **aggregate** demand affect **labor** demand

- $w_{t+m}(i) \equiv \frac{W_{t+m}(i)}{P_{i,t+m}}$; $A \equiv \left(\alpha \frac{\eta-1}{\eta}\right)^{\frac{\eta}{\alpha+\eta-\alpha\eta}}$; $f_{t+m}(.) \rightarrow$ discounting and weighting function of aggregates.

Dynamic Labor Supply (Implicit Function)

$$log(L_{it}) = \frac{1}{\varphi} log(w_t(i)) + \frac{1}{\varphi} log(C_{it}^{-\sigma}) + log(\chi)$$

$$C_{it}^{-\sigma} = \frac{\beta}{Q_{t,t+1}} \mathbb{E}_t \left\{ \frac{P_{it}}{P_{i,t+1}} C_{i,t+k}{}^{-\sigma} \right\}$$

$$C_{it} + Q_{t,t+1} \frac{B_{i,t+1}}{P_{it}} = w_t(i) L_{it} + \frac{B_{it}}{P_{it}} + \frac{D_{it}}{P_{it}} - \frac{T_t(i)}{P_{it}}$$

Dynamic Labor Supply (Implicit Function)

$$log(L_{it}) = \frac{1}{\varphi} log(w_t(i)) + \frac{1}{\varphi} log(C_{it}^{-\sigma}) + log(\chi)$$

Labor supply shifter

$$C_{it}^{-\sigma} = \frac{\beta}{Q_{t,t+1}} \mathbb{E}_t \left\{ \frac{P_{it}}{P_{i,t+1}} C_{i,t+k}{}^{-\sigma} \right\}$$

Substitution effects (real interest rate)

$$C_{it} + Q_{t,t+1} \frac{B_{i,t+1}}{P_{it}} = w_t(i) L_{it} + \frac{B_{it}}{P_{it}} + \frac{D_{it}}{P_{it}} - \frac{T_t(i)}{P_{it}}$$

Wealth effects

Conventional approach:

- Demand and supply **curves**
- L_t^D and L_t^S functions of w_t
- Demand-driven shifts in expectations of w_{t+k} are shifters *of* the supply curve

Our approach:

- Demand and supply **planes**
- L_{t+k}^D and L_{t+k}^S functions of w_{t+k} in **sequence space** (as in Auclert et al (2021, 2024))
- Demand-driven shifts in expectations of w_{t+k} are shifters *along* the supply plane

Claim #1

Estimating $\frac{dL}{dG}$ in the cross section

Cross-sectional estimates isolate labor demand response to G shocks

Assumptions:

1. Federal taxation $\leftrightarrow$ real taxes equal across regions

   Our take: weak/reasonable assumption in fiscal union

2. No home bias in consumption

   Our take: strong assumption, but clear guidance on what to control for

Claim #1: Sketch of Proof

$$log(L_{it}) = \frac{1}{\varphi} log(w_t(i)) + \frac{1}{\varphi} log(C_{it}^{-\sigma}) + log(\chi)$$

$$C_{it}^{-\sigma} = \lim_{k \to \infty} \mathbb{E}_t \left\{ C_{i,t+k}^{-\sigma} \right\}$$

Long run Euler equation: no relative substitution effects in currency union

$$C_{it} + Q_{t,t+1} \frac{B_{i,t+1}}{P_{it}} = w_t(i) L_{it} + \frac{B_{it}}{P_{it}} + \frac{D_{it}}{P_{it}} - \frac{T_t(i)}{P_{it}}$$

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Assumption 1: No differential taxation across regions

Claim #1: Sketch of Proof

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Assumption 2: No differential asset deflation

Claim #1: Sketch of Proof

$$log(L_{it}) = \frac{1}{\varphi} log(w_t(i)) + \frac{1}{\varphi} log(C_{it}^{-\sigma}) + log(\chi)$$

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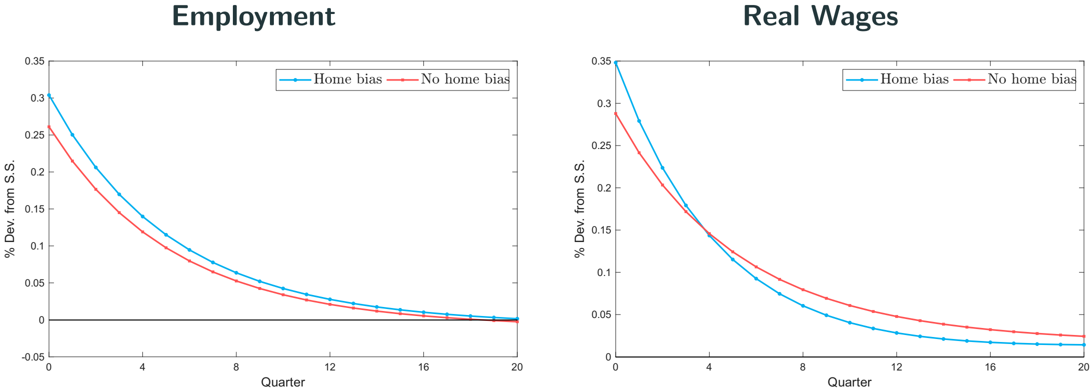
Home bias? $\rightarrow$ Control for asset deflation

Dif-in-dif responses with no / 85% home bias



Upper bounds on bias: home bias also leads to labor **demand** effects in same direction

Claim #2

Calibrated sufficient statistic for labor supply effects of *expected* taxation

$$w_t\left(i\right)\frac{\partial L_{it}^{S}}{\partial T\left(i\right)}=\left[1+\frac{C_{it}}{w_t\left(i\right)L_{it}}\frac{\varphi}{\sigma}\right]^{-1}\frac{\partial}{\partial T\left(i\right)}\left(Q_{t,t+1}\frac{B_{i,t+1}}{P_{it}}\right).$$

Labor supply ⇔ Savings

Calibrated sufficient statistic for labor supply effects of *expected* taxation

$$w_t(i)\frac{\partial L_{it}^S}{\partial T(i)} = \left[1 + \frac{C_{it}}{w_t(i)L_{it}}\frac{\varphi}{\sigma}\right]^{-1}\frac{\partial}{\partial T(i)}\left(Q_{t,t+1}\frac{B_{i,t+1}}{P_{it}}\right).$$

Light calibration:

- Two structural parameters: φ and σ
- One simple moment: Ratio of consumption to labor income

Claim #2: Empirical Approach

Challenge: Differential expectations of taxes across regions unobservable.

Our approach:

- Identified G shocks + Claim 1 $\Rightarrow$ estimated L and B responses isolate labor **demand**
- $\Rightarrow$ residual variation in L and B are gives and indication of labor supply's role in remaining employment variation.

1. Theory: Fiscal Policy in a Currency Union

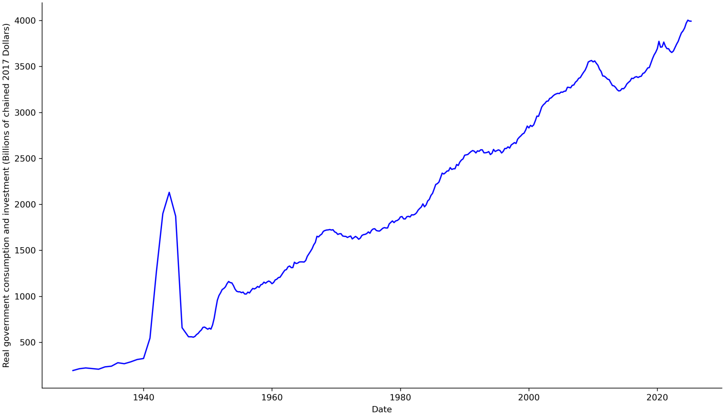
2. New archival data: WWII labor markets

3. Empirical strategy

4. Estimating labor demand and supply

World War II:

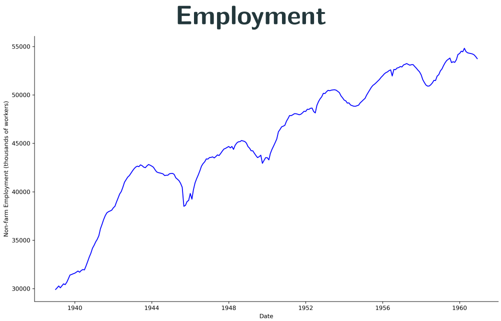
- Largest shock to government spending in US history



Why World War II?

World War II:

- Largest shock to government spending in US history
- Leads to enormous cyclical increase in employment



Why World War II?

<u>World War II</u>:

- Largest shock to government spending in US history
- Leads to enormous cyclical increase in employment
  - With obvious labor demand and supply components
- In a data-rich environment

Data Overview

War Production Board Form 732 [Digitized from National Archives]:

- Survey of the 1,779 largest manufacturing plants
- **Number of employees, hours per employee, sales, hourly wages, gender, backlogs**
- Commuting zone × industry × quarter

BLS Bulletin No. 966 [Digitized from online materials]:

- **CPI** for 72 cities
- Spatially interpolated to cover all commuting zones

Annual Treasury Department bulletins [Digitized from online materials]:

- **War bond** purchases by State
- State × month

War Manpower Commission form E-270 [Digitized from National Archives]:

- **Employment, quits, layoffs, accessions, USES placements**
- Commuting zone × month

War Manpower Commission Inter-county migration [Digitized from National Archives]:

- **Population**
- Commuting zone in two months during the war

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Empirical Approach

Estimating effects of G on employment and wages

- **Two-way fixed effects** eliminate "attractive labor markets" or national productivity cycles.
- **Remaining challenge:** G may be directed to labor markets when they're booming.

(Leave one out) Shift-Share Instrument

- Predict **current** public procurement to city c G_{ct} via:
  - City c's industrial structure S_{ic} at **beginning of war** interacted with
  - **national** demand for goods of industry i.

$$\ln(Z_{c,t}) = \ln\left(\sum_i [S_{ic} \times G_{it}] - G_{cit}\right)$$

- Assumption: **National** G shifts across industries due to strategic war needs, not national productivity in industry i.
  - See Ilzetzki (2024) for historical discussion

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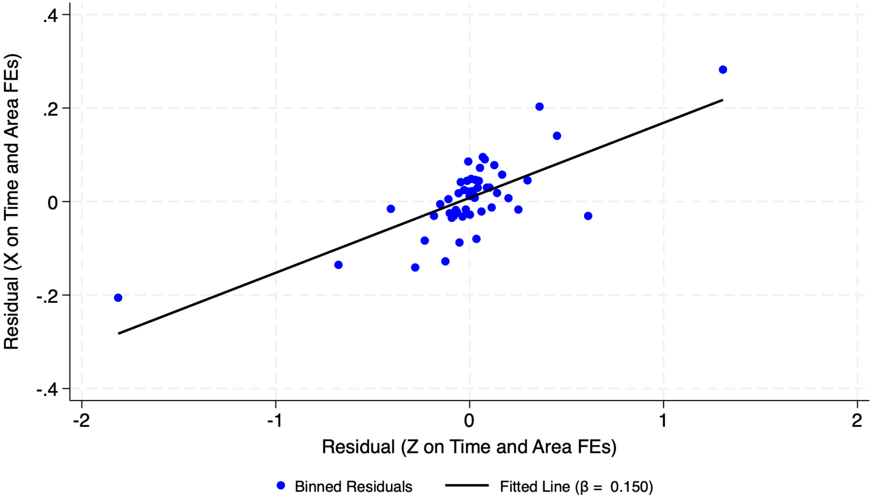
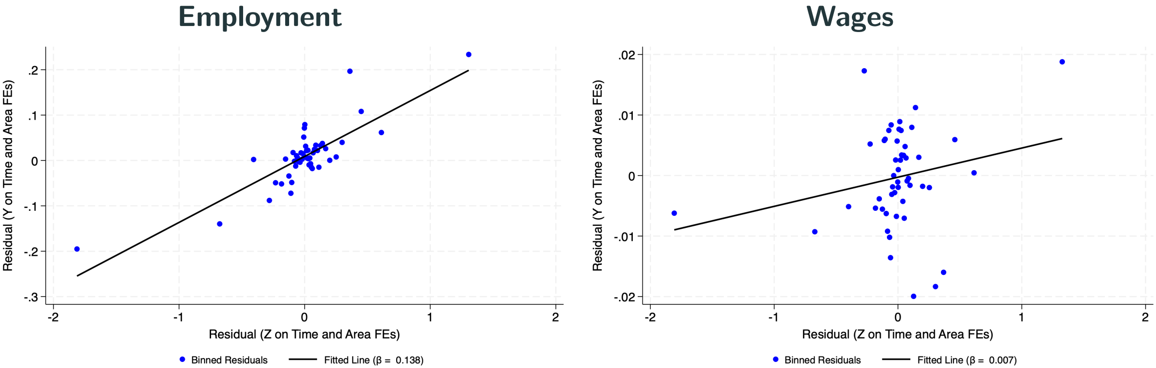
First Stage



Residual (X on Time and Area FEs)
.4
.2
0
-.2
-.4
-2
-1
0
1
2
Residual (Z on Time and Area FEs)
Binned Residuals
Fitted Line (β = 0.150)

Reduced-form regressions $\frac{\partial Y}{\partial z}$



Effects of G on Labor Demand and Wages

Treatment variable: $log(G_{ct})$

Outcome variable:	**log(Employment)**				**log(wages)**			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	0.92***	0.92***	0.99***	1.00***	0.14***	0.14***	0.03	0.06**
	(0.02)	(0.02)	(0.14)	(0.15)	(0.01)	(0.01)	(0.03)	(0.03)
Quarter FE		Yes		Yes		Yes		Yes
City FE			Yes	Yes			Yes	Yes
F-Stat	226	195	18	16	190	189	37	31
Observations	1,484	1,484	1,484	1,484	1,380	1,380	1,380	1,380

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

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~\$3.7K per job ≈ \$45K in 2008

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Compare with ~$35K in Chodorow Reich (2019)

Labor Demand: It's all the extensive margin

Treatment variable: $log(G_{ct})$

	Total Weekly Hours				Average Hours per Worker			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	0.92***	0.93***	0.96***	0.99***	0.00	0.01*	-0.04	-0.03
	(0.02)	(0.02)	(0.14)	(0.15)	(0.00)	(0.00)	(0.03)	(0.03)
Quarter FE		Yes		Yes		Yes		Yes
City FE			Yes	Yes			Yes	Yes
F-Stat	225	195	18	16	225	195	18	16
Observations	1,483	1,483	1,483	1,483	1,483	1,483	1,483	1,483

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Peak US government procurement + construction = \$62 billion vs. \$7 billion in 1941

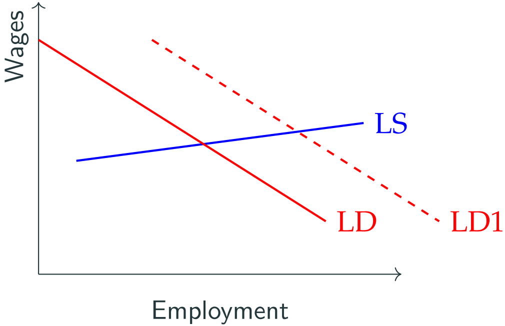
- Our estimates ΔEmployment from labor demand → 14 million

Actual job growth 1941 to peak: 21 million

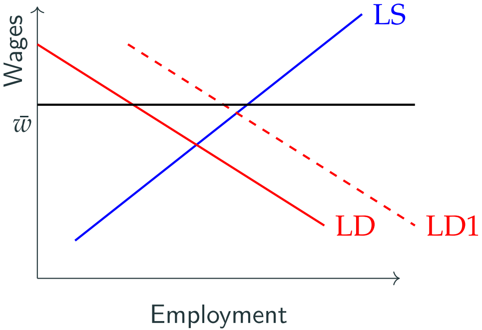
- 8 million growth of US non-farm, non military employment
- + 13 million peak armed forces to replace

⇒ **US WWII employment growth: $\frac{2}{3}$ labor demand, $\frac{1}{3}$ other factors incl. labor supply**

Large labor and small wage response to labor demand shock → **Highly elastic labor supply**

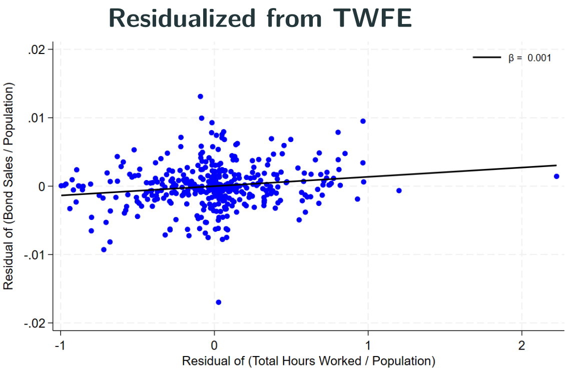


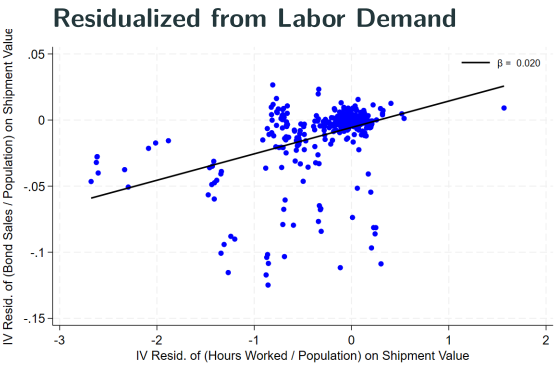
Implied Frisch elasticity of labor supply: $\varphi = 17$**!**



This implies NO ROLE for labor supply

Bond purchases vs. Employment





Inconsistent with ANY parameterization of **Claim 2**

Labor supply can't explain residual cross-sectional **employment** variation

- Still leaves missing intercept: Perhaps no cross-sectional variation in expected taxes.

Next directions

Sufficient statistics in sticky wage model

Exploring labor market frictions and mobility, using new data

Exploring non-pecuniary labor supply motivations

- Patriotism
- Instrumental feminism / patriotic feminism

Appendix

Labor Demand Problem

Back

Price setting problem is

$$\max_{p_t(i,j)} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[p_t(i,j) y_{t+k|t}(i,j) - W_{t+k}(i) \ell_{t+k}(i,j) \right] \right\}$$

$$\text{s.t.} \quad y_{t+k}(i,j) = \ell_{t+k}(i,j)^{\alpha} \quad \text{and} \quad y_{t+k|t}(i,j) = \left(\frac{p_t(i,j)}{P_{t+k}(i)} \right)^{-\eta} Y_{t+k}(i).$$

Can combine constraints to obtain inverse product demand as function of $\ell_{t+k}(i,j)$

$$p_t(i,j) = \left(\frac{\ell_{t+k}(i,j)^{\alpha}}{Y_{t+k}(i)} \right)^{-\frac{1}{\eta}} P_{t+k}(i).$$

Since price $p_t(i,j)$ is fixed across time, labor demands for any two periods $t+k$ and $t+h$ are linked by

$$\left(\frac{\ell_{t+k}(i,j)^{\alpha}}{Y_{t+k}(i)} \right)^{-\frac{1}{\eta}} P_{t+k}(i) = p_t(i,j) = \left(\frac{\ell_{t+h}(i,j)^{\alpha}}{Y_{t+h}(i)} \right)^{-\frac{1}{\eta}} P_{t+h}(i), \quad \forall k, h \geq 0.$$

Can write this as a law-of-motion for labor demand

$$\ell_{t+k+1}(i,j) = \ell_{t+k}(i,j) \left(\frac{Y_{t+k+1}(i)}{Y_{t+k}(i)} \right)^{\frac{1}{\alpha}} \left(\frac{P_{t+k+1}(i)}{P_{t+k}(i)} \right)^{\frac{\eta}{\alpha}}.$$

Leading to final problem setup

$$\max_{\{\ell_{t+k}(i,j)\}_{k=0}^{\infty}} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[p_t(\ell_{t+k}(i,j)) \, y_{t+k|t}(\ell_{t+k}(i,j)) - W_{t+k}(i) \ell_{t+k}(i,j) \right] \right\}$$

$$\text{s.t.} \quad \ell_{t+k+1}(i,j) = \ell_{t+k}(i,j) \left(\frac{Y_{t+k+1}(i)}{Y_{t+k}(i)} \right)^{\frac{1}{\alpha}} \left(\frac{P_{t+k+1}(i)}{P_{t+k}(i)} \right)^{\frac{\eta}{\alpha}}.$$

Full Characterization of Firm Labor Demands

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Price-adjusting firms, when choosing a price at time t, are not only implicitly pinning down a contemporaneous labor demand $\ell_t(i,j)$, but a full sequence of labor demands $\{\ell_{t+k}(i,j)\}_{k=0}^{\infty}$. The more generic solution to the maximization problem is then

$$\ell_{t+k}(i,j) = A\,\mathbb{E}_t\left\{Y_{t+k}(i)^{\frac{1}{\alpha+\eta-\alpha\eta}}\left(\frac{P_{t+k}(i)}{P_{i,t+k}}\right)^{\frac{\eta}{\alpha+\eta-\alpha\eta}}\right\}\left(\mathbb{E}_t\left\{\sum_{m=0}^{\infty} f_{k+m}(\theta,Q,Y,P)\frac{P_{i,t+k+m}}{P_{i,t+k}}\,w_{t+k+m}(i)\right\}\right)^{-\frac{\eta}{\alpha+\eta-\alpha\eta}}.$$

- $A \equiv \left(\alpha\frac{\eta-1}{\eta}\right)^{\frac{\eta}{\alpha+\eta-\alpha\eta}}$; $\qquad f_{k+m}(\theta,Q,Y,P) \equiv \dfrac{\theta^{k+m}Q_{t,t+k+m}\left(\frac{Y_{t+k+m}(i)}{Y_{t+k}(i)}\left(\frac{P_{t+k+m}(i)}{P_{t+k}(i)}\right)^{\eta}\right)^{\frac{1}{\alpha}}}{\mathbb{E}_t\left\{\sum_{h=0}^{\infty}\theta^{k+h}Q_{t,t+k+h}\frac{Y_{t+k+h}(i)}{Y_{t+k}(i)}\left(\frac{P_{t+k+h}(i)}{P_{t+k}(i)}\right)^{\eta}\right\}}.$

Non-adjusting firms take their previously chosen price $p_t(i,j)$ as given and labor demand satisfies product demand at their set price, conditional on making non-negative profits.

$$\ell_t(i,j) = \begin{cases}\left(\frac{p_t(i,j)}{P_t(i)}\right)^{-\frac{\eta}{\alpha}} Y_t(i)^{\frac{1}{\alpha}}, & \text{if} \quad w_t(i) \le \frac{p_t(i,j)}{P_{it}}\left(\frac{p_t(i,j)}{P_t(i)}\right)^{\frac{\eta(1-\alpha)}{\alpha}} Y_t(i)^{-\frac{1-\alpha}{\alpha}} \\ \left(\frac{p_t(i,j)}{P_{it}}\right)^{\frac{1}{1-\alpha}} w_t(i)^{-\frac{1}{1-\alpha}}, & \text{otherwise.}\end{cases}$$

Baseline Calibration

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Parameter	Interpretation	Value
β	Discount factor	0.99
σ	Inverse EIS	1
φ	Inverse Frisch elasticity	1
χ	Disutility of labor shifter	0.8
$\omega_i(i)$	Home preference	2/3
γ	Elasticity of demand across regions	1
ρ_g	Autocorrelation of government spending	0.9
$\bar{S}_G$	Steady-state government share of output	0.4
ρ_i	Autocorrelation Taylor rule	0.99
ϕ_π	Inflation feedback Taylor rule	1.5
ϕ_y	Output feedback Taylor rule	0.125
η	Elasticity of demand across varieties	9
α	Labor share	0.7
θ	Degree of price rigidity	0.75
$\tau(i)$	Regional tax burden	0.5