ZIPF'S LAW FOR CURRENCIES

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Overview

EVIDENCE OF A SIMILAR INVERSE RANK-SIZE RELATIONSHIP in many social science contexts: **Zipf's law**

ZIPF'S LAW FOR INTERNATIONAL CURRENCIES

- Document an *even more* skewed distribution than in other settings: **Hyper-Zipf** distribution
- By some metrics, USD and EUR are outsized even relative to this skewed distribution
- Using comprehensive data on transactions: SWIFT

US UNIQUE: Nearly 100% of US transactions conducted in USD

- Dollar use percolates through US trading partners into the international system

NETWORK THEORY to contextualize these findings

Academic Research Context

Dollar dominance
Boz et al (2021); Eichengreen (1991); Gopinath (2015); Gopinath and Itzhoki (2010); Ilzetzki, Reinhart and Rogoff (2019, 2020a, 2002b, 2021); Gopinath and Stein (2019); Maggiori, Neiman and Schreger (2020); Rey(2013)

Zipf's Law

- See Gabaix (2010) for a review
- Holds for phenomena in economics, other social sciences, biology, and more.
- Literature review to follow

Outline

1. Documenting Zipf's Law for International Currencies
2. Dissecting dollar dominance: The importance of US transactions in the dollar's global use
3. Network theory $\rightarrow$ Zipf's Law

Zipf's Law for International Currencies

Data

Society for Worldwide Interbank Financial Telecommunication, **SWIFT** is a cooperative enabling financial institutions to send messages on payment orders.

- Handles ~30 million messages (transfers) a day (in 2021)
- ~50% of all global transfers

Our dataset

- Monthly from 2011-2017
- Message type MT103
  - All transfers with at least one non-bank party more
- Panel of volume and value of messages by {**o**rigin, **d**estination, **c**urrency, **m**onth}
- Drop {o,d,c} with <12 transactions a year, c = VEF
- 700 million messages

Zipf's Law

Obs. drawn from a size power law distribution of the form

$$f(x) = \alpha / x^{-(1+\zeta)}$$

will have a rank-size relationship

$$\ln(\textit{rank}) = -\zeta \ln(\textit{size}) + \textit{constant}$$

Zipf's Law refers to $\zeta = 1$

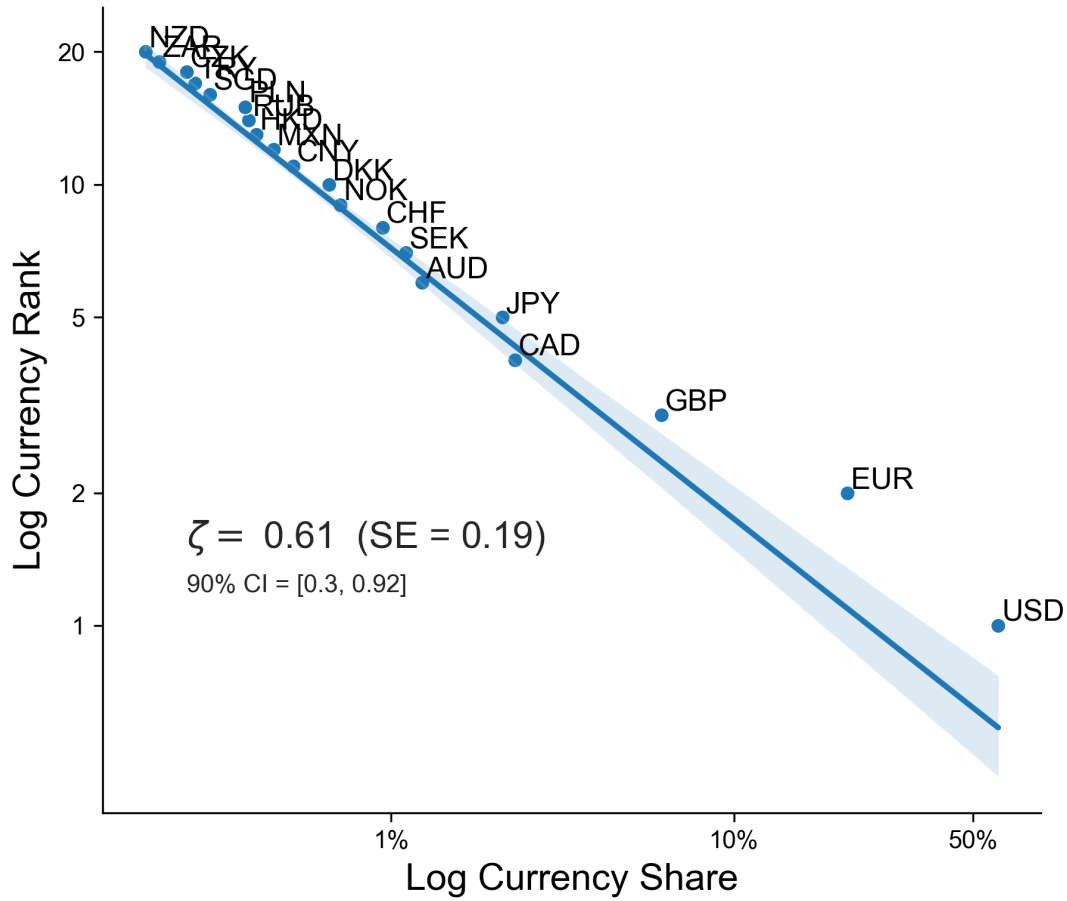
- Characterizes many empirical populations in social science

  Literature
- Implies biggest city (e.g.) $2\times$ larger than 2^{nd}, $3\times$ 3^{rd}...

We find a more unequal relationship $\zeta < 1 \Rightarrow$ **Hyper-Zipf**

- For currencies $\zeta \approx 0.6$
- Largest currency (USD) $2^{\frac{1}{0.6}}\times$ larger than 2^{nd}, $3^{\frac{1}{0.6}}\times$ 3^{rd}...

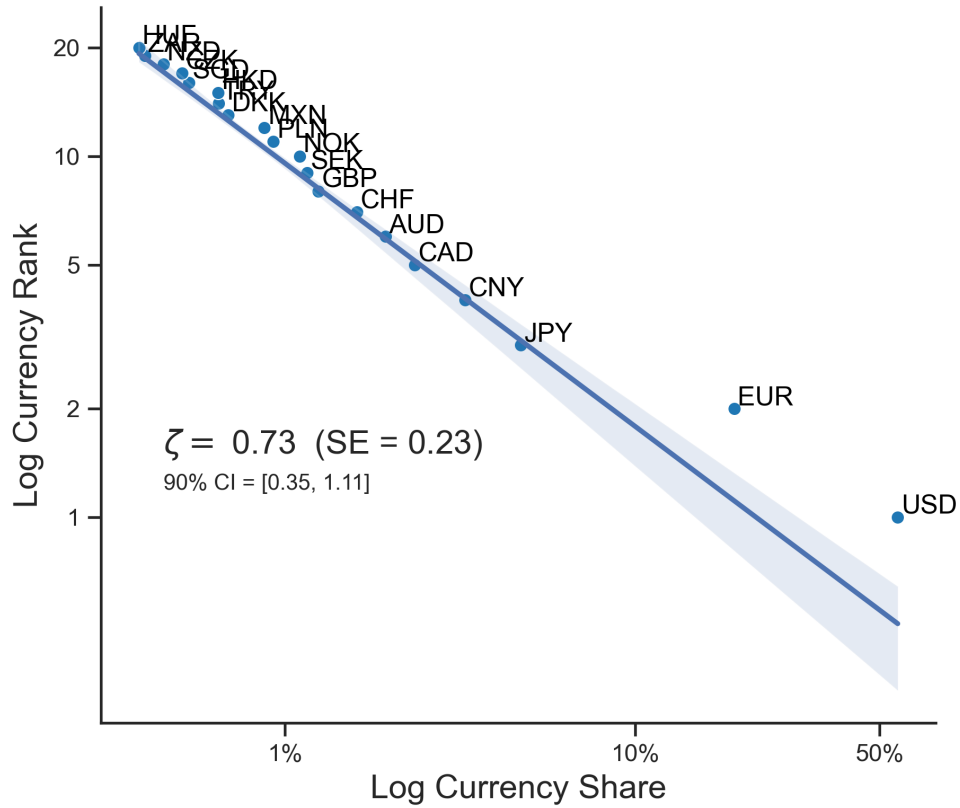
Zipf's Law for International Currencies



All Currencies | KS Test | Exponential | Log-normal | Include intra-EZChina | Include Domestic

All intl. transactions ex. intra-EZ and intra-China. Top-20 currencies. Line represents OLS log-log regression with Gabiax-Ibragimov bias correction; 95% confidence interval is shaded. Slope of the regression and small-sample-adjusted standard errors reported. Sources: SWIFT and the authors.

Zipf's Law for Vehicle Transactions

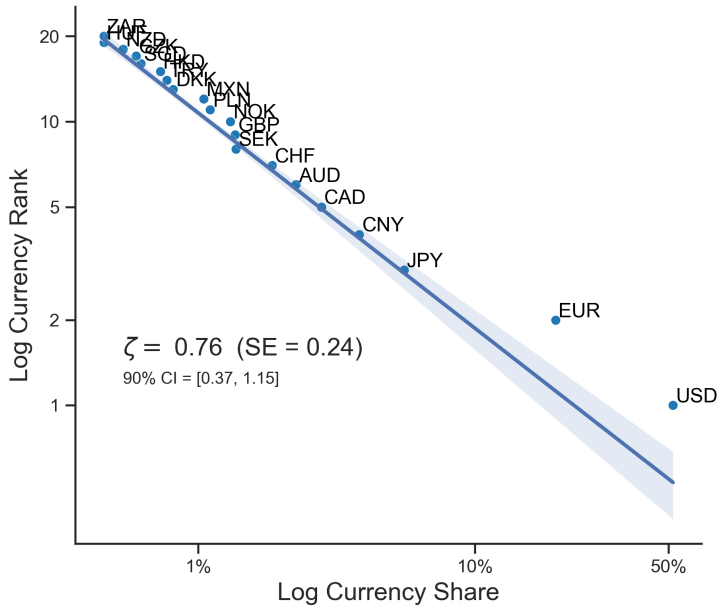


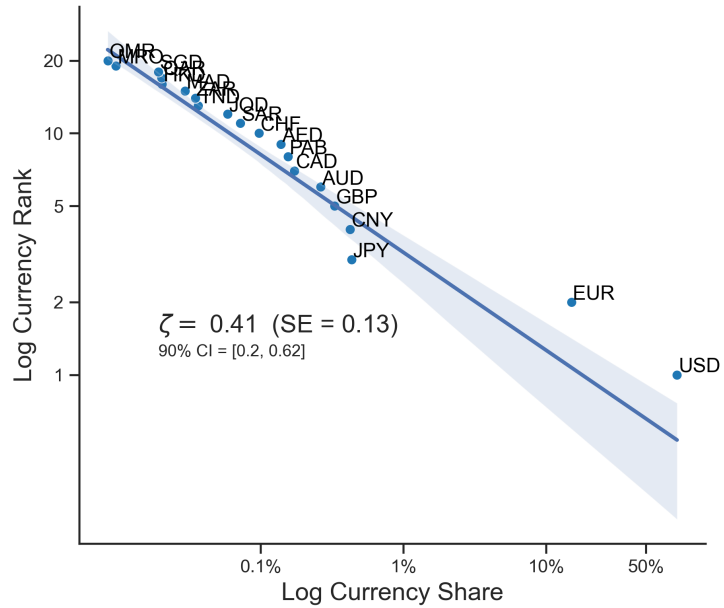
All Currencies | KS Test | Exponential | Log-normal

All intl. transactions ex. intra-EZ and intra-China, where the currency isn't the national currency of origin or of destination country (vehicle transactions). Top-20 currencies. Line represents OLS log-log regression with Gabiax-Ibragimov bias correction; 95% confidence interval is shaded. Slope of the regression and small-sample-adjusted standard errors reported. Sources: SWIFT and the authors.

Zipf's Law for Vehicle Transactions

High income (left) and Emerging Market (right) Economies

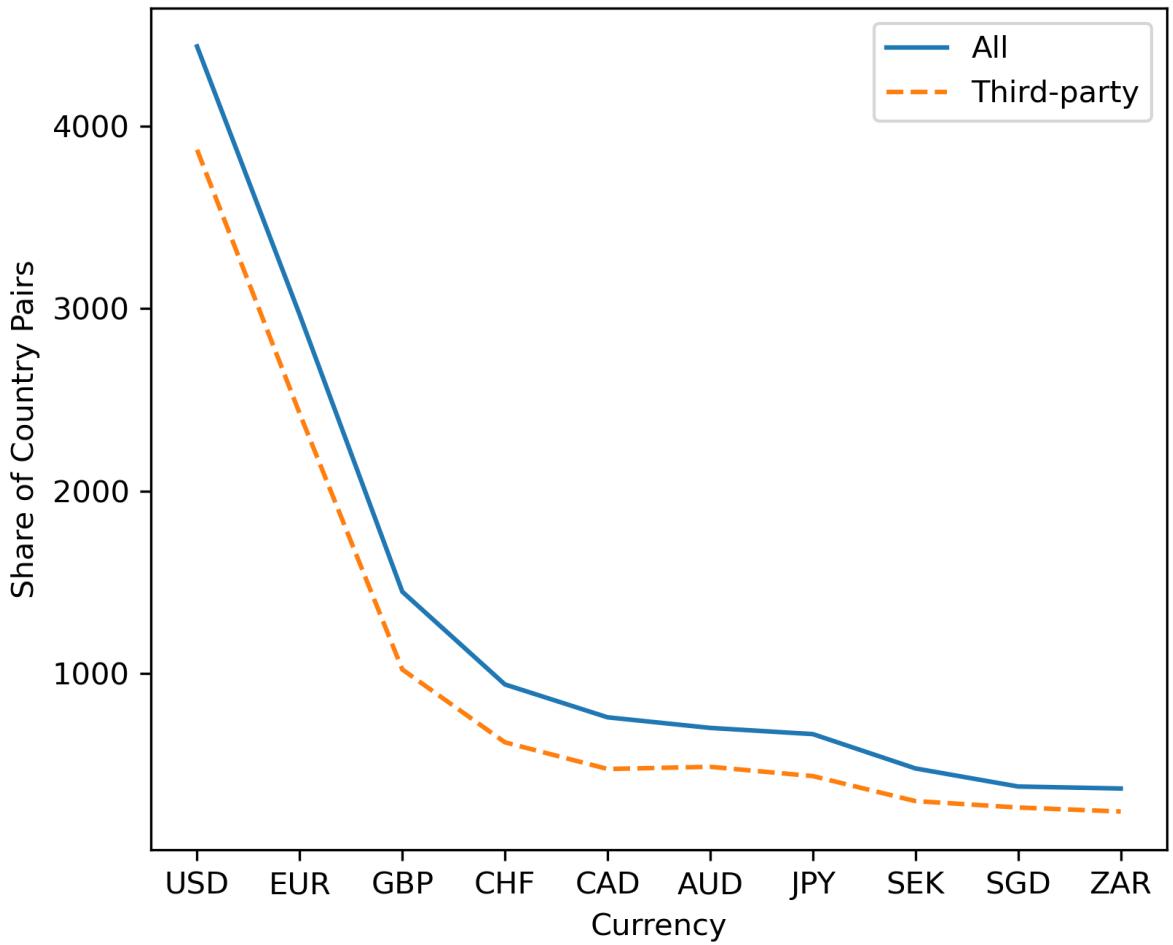




All intl. transactions ex. intra-EZ and intra-China, where the currency isn't the national currency of origin or of destination country (vehicle transactions). Top-20 currencies. Line represents OLS log-log regression with Gabiax-Ibragimov bias correction; 95% confidence interval is shaded. Slope of the regression and small-sample-adjusted standard errors reported. Sources: SWIFT and the authors.

Few Currencies Used Universally

Number of country pairs with *any* transactions in each currency



Sources: SWIFT and the authors

Hyper-Zipf in International Currencies

Power law distribution with $\zeta < 1 \Rightarrow$ **Hyper-Zipf**

USD (and possibly EUR) are overweight even relative to this distribution

Also holds for

- Cryptocurrencies Crypto

Isn't explained by / doesn't hold for

- Government reserve holdings Reserves
- Corporate bond denomination (Maggiori, Neiman and Schreger 2020) Bonds
- Country GDP GDP
- Goods and Services Trade Trade
- SWIFT messages by *country* ValueVolume

Dissecting Dollar Dominance

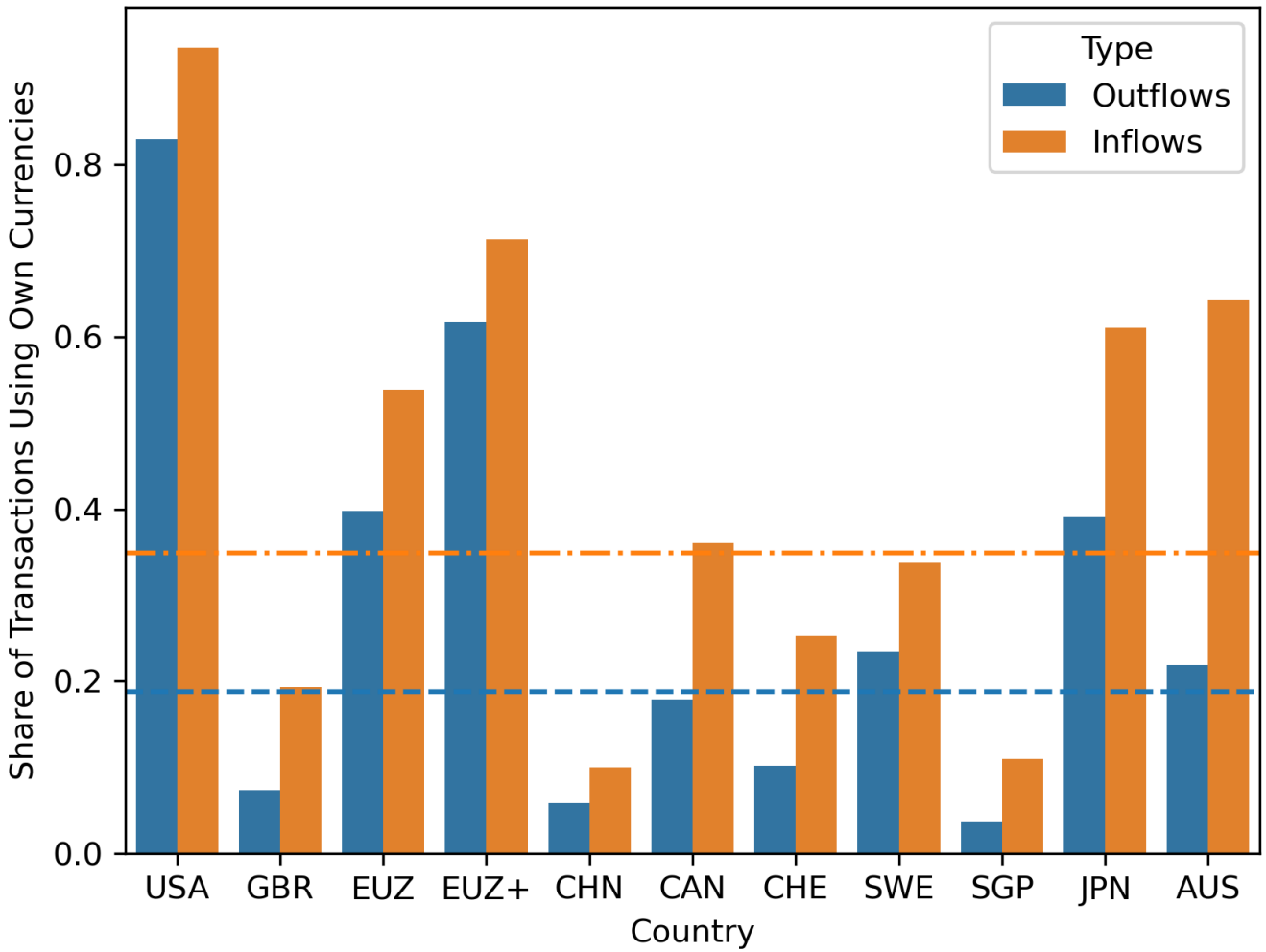
Summary Statistics

Share of World Transactions (by value):

	All Transactions	Ex-US and Euro Area
Origin Currency	32.8%	18.7%
Destination Currency	53.4%	34.8%
Vehicle Currency	14.2%	48.1%
of which USD	60.0%	64.6%
of which EUR	19.0%	25.7%

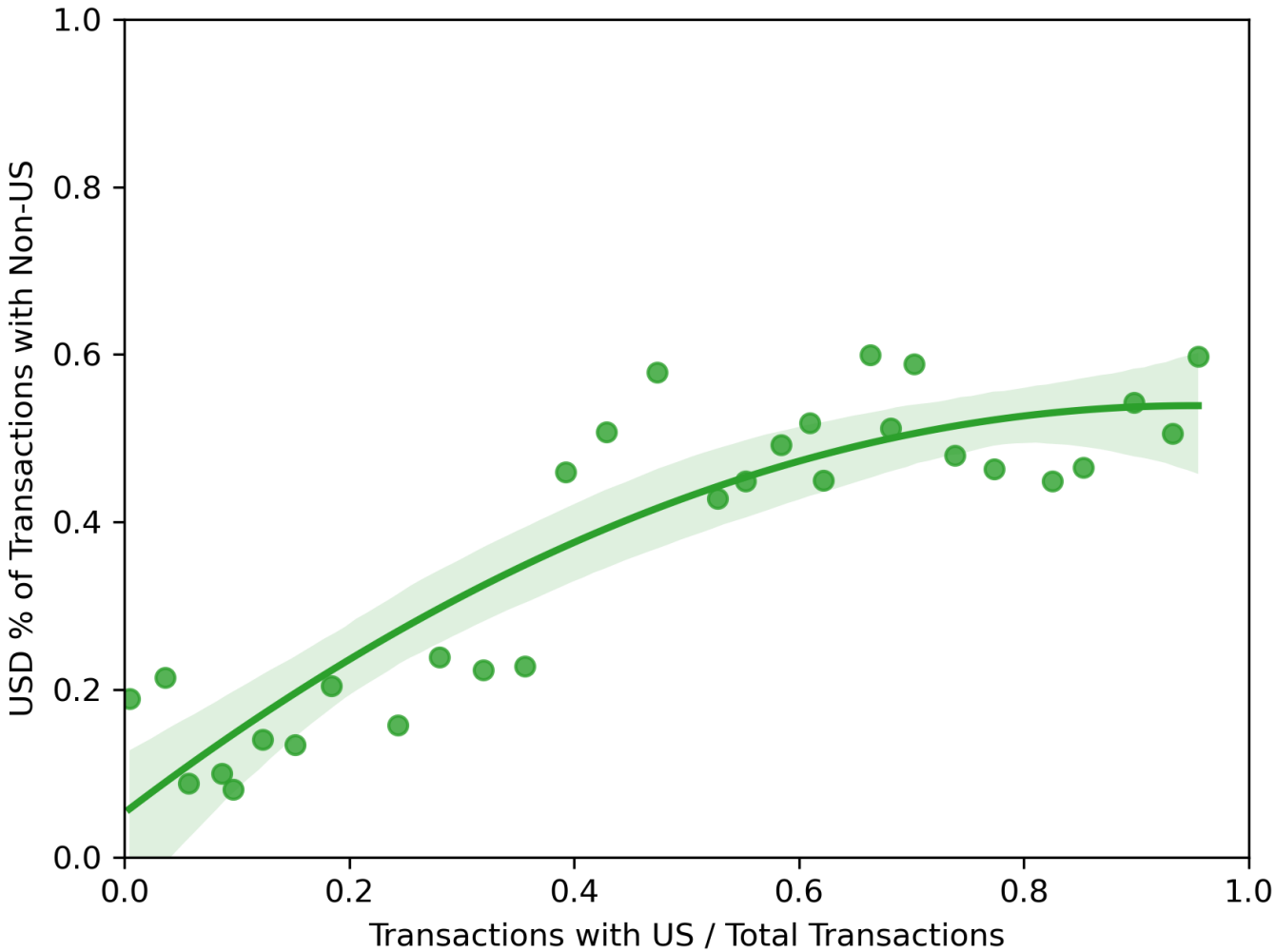
Source: SWIFT and the authors

US Unique in Own-Currency Denomination



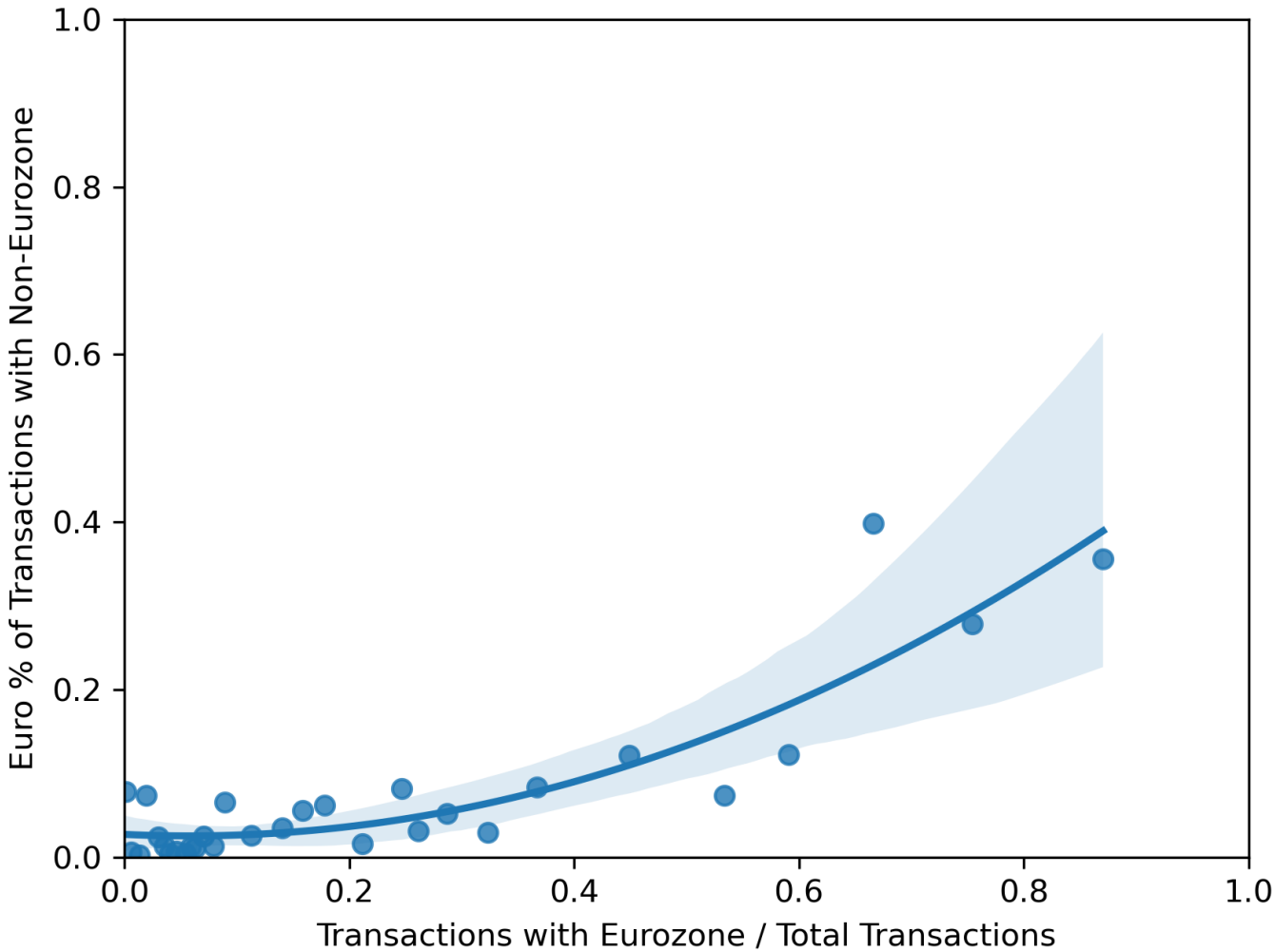
Share of inbound (orange) and outbound (blue) transactions denominated in the own currency. Top 10 countries ranked from left to right by flow value. EUZ excludes and EUZ+ includes intra-Eurozone flows. Dashed lines show averages for non-US countries in this sample. Sources: SWIFT and the authors

Trading with the US leads to Dollar Trade



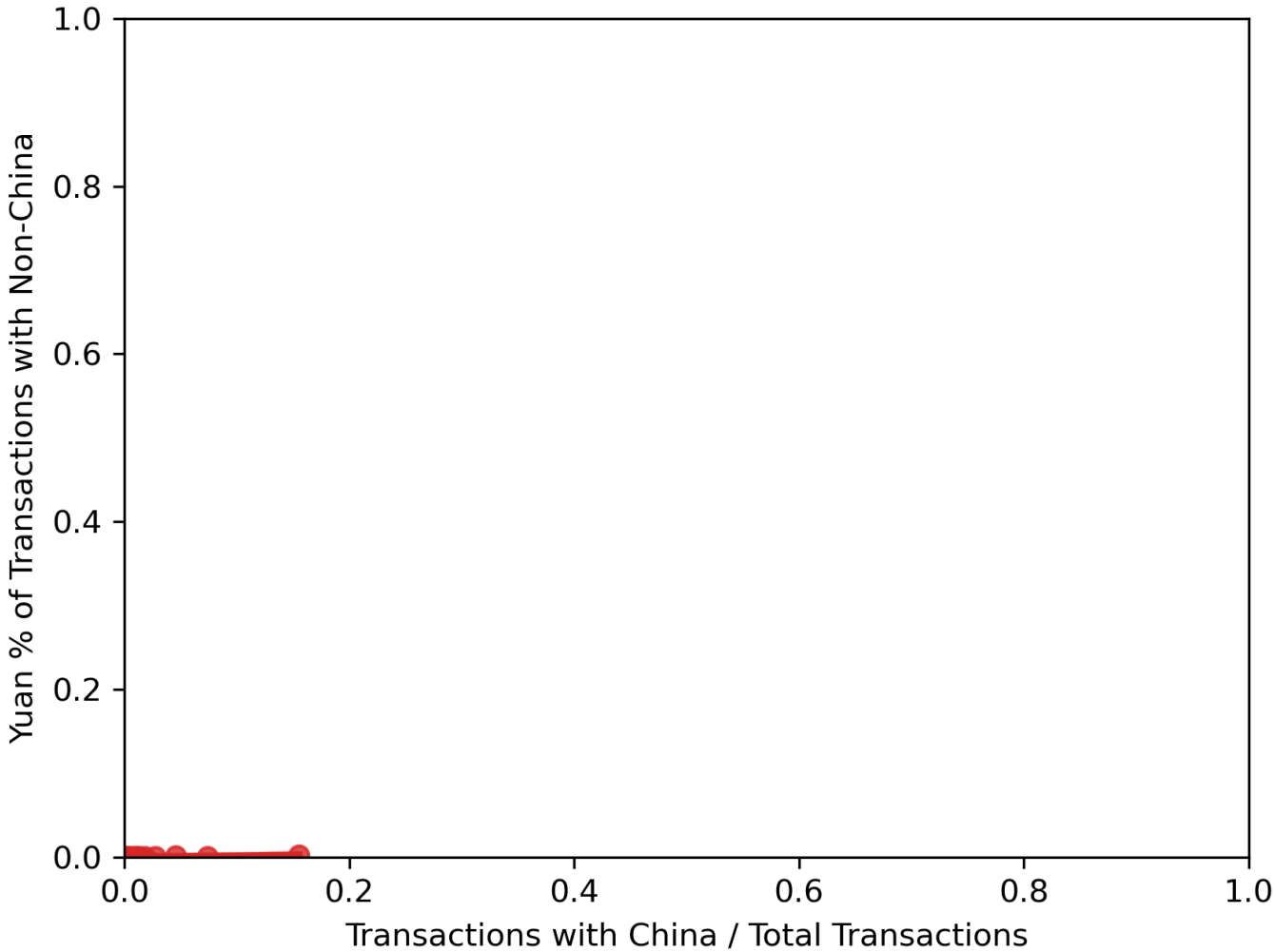
Bin scatter plot. Percent of a country's transactions with countries ex-US that are denominated in USD against share of total transactions of that country that are conducted with the US. Quadratic fit in line and 95% confidence interval in shaded area. Sources: SWIFT and the authors

Euro as a Regional Currency



Bin scatter plot. Percent of a country's transactions with countries ex-EZ that are denominated in EUR against share of total transactions of that country that are conducted with the Eurozone. Quadratic fit in line and 95% confidence interval in shaded area. Sources: SWIFT and the authors

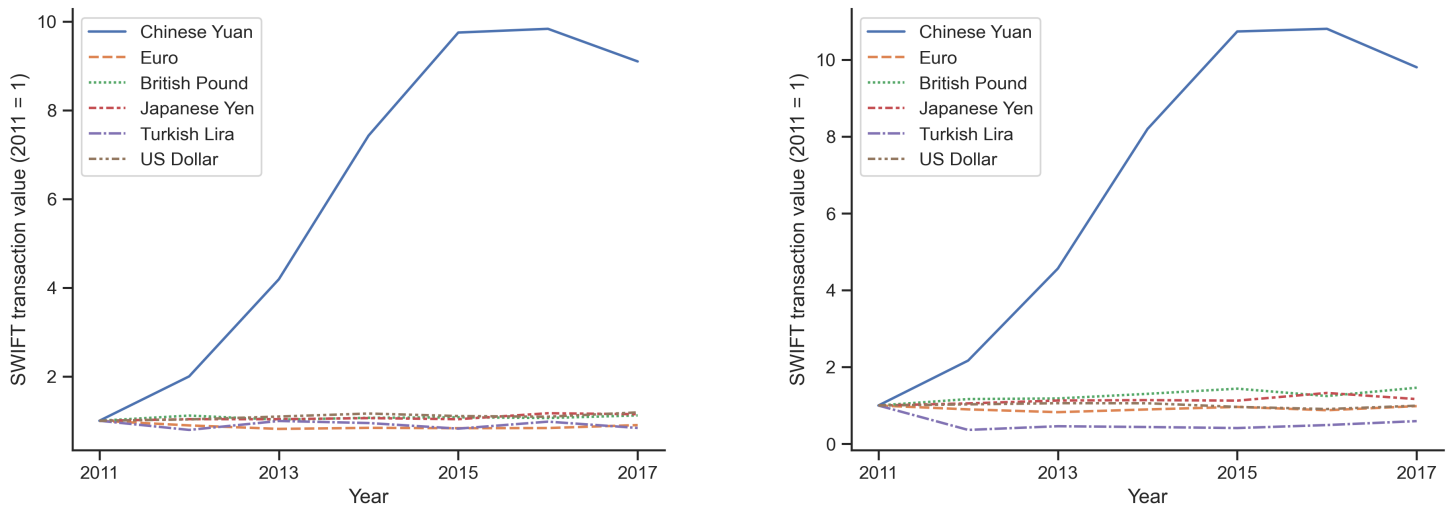
Renminbi as a Local Currency



Bin scatter plot. Percent of a country's transactions with countries ex-China that are denominated in RMB against share of total transactions of that country that are conducted with the China. Quadratic fit in line and 95% confidence interval in shaded area. Sources: SWIFT and the authors

Rise of the Reniminbi

All (left) and 3rd Party (right) Transactions



Volume of SWIFT transactions conducted in major currencies. 2011 = 1. Left hand panel: all transactions. Right hand panel: Transactions where neither party in the transaction is in country issuing the currency (3rd party). Sources: SWIFT and the authors.

Decomposing Currency Choice

Dependent variable: share of currency *c* in transactions from origin *o* to destination *d*, by value							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Currency % in Origin	0.252	0.207	0.248	0.217	0.352	0.346	0.355
	(0.009)	(0.010)	(0.009)	(0.010)	(0.011)	(0.011)	(0.010)
Currency % in Destination	0.674	0.682	0.596	0.611	0.711	0.757	0.716
	(0.009)	(0.010)	(0.009)	(0.010)	(0.011)	(0.012)	(0.011)
Origin Currency?			0.108	0.092	0.089	0.067	0.067
			(0.004)	(0.004)	(0.004)	(0.004)	(0.003)
Destination Currency?			0.198	0.195	0.180	0.170	0.157
			(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
USD?					-0.060	-0.150	-0.112
					(0.011)	(0.012)	(0.011)
Euro?					0.012	-0.015	-0.007
					(0.005)	(0.005)	(0.005)
World Share of Currency					-0.162	-0.094	-0.132
					(0.016)	(0.018)	(0.016)
USD $\times$ USA							0.366
							(0.011)
Euro $\times$ Euro Area							0.301
							(0.015)
Observations	811,509	706,384	811,509	706,384	811,509	706,384	811,509
Adj. R^2	0.51	0.50	0.57	0.55	0.58	0.56	0.60
Incl. US and Euro Area	YES	NO	YES	NO	YES	NO	YES

Standard errors in parentheses

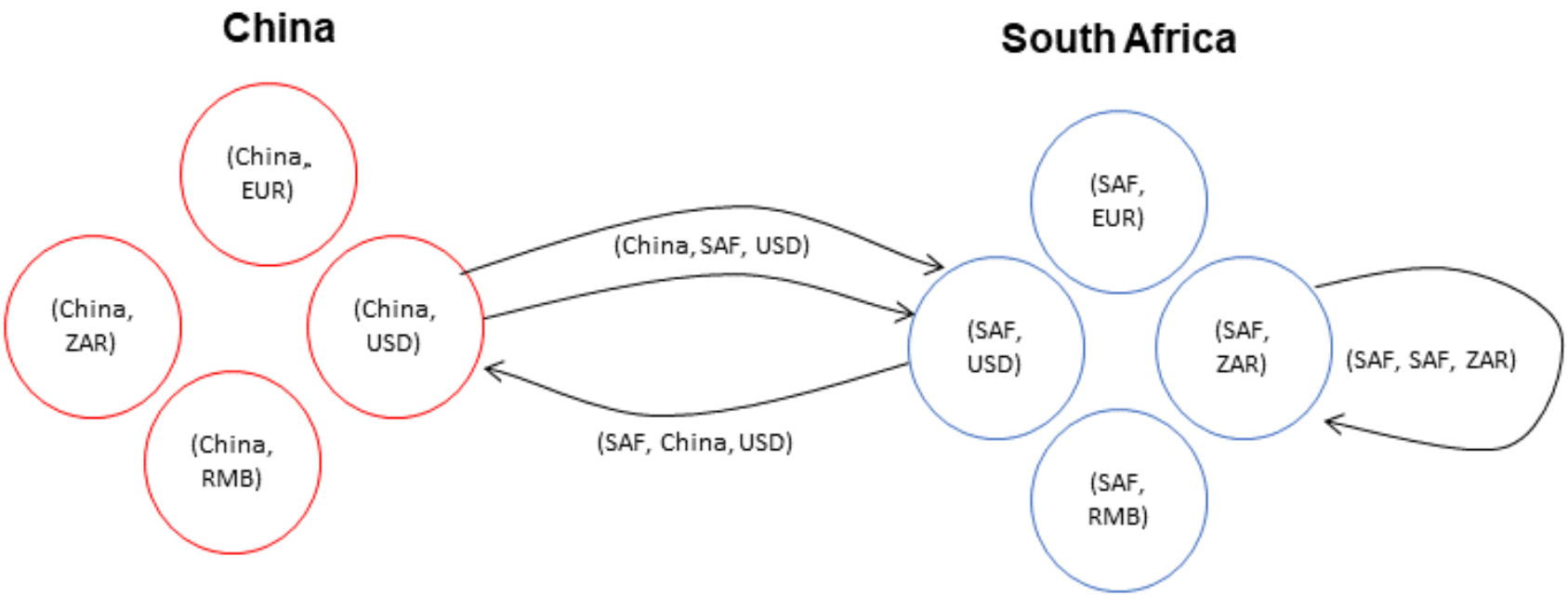
Summary: Empirical Findings

- Currency choice follows a power law with $\zeta < 1$: Outsized role for top currencies.
- Currency choice in a origin-destination country pair governed by currencies' popularity in the two countries.
  - PPCP: Producer preferred currency pricing is more common than
  - LPCP: Local preferred currency pricing
  - Additional role for producer and local currencies (PCP and LCP)
- No additional role for large currencies, including \$ and €
- *Except* for transactions with the US and the Euro Area

The Network of Intl. Currencies (Preliminary)

Setup

- N countries indexed by i
- C distinct currencies
- Time t is discrete
- Nodes denoted by (i, c)
- Directed edges represented by (o, d, c)
- Each edge represents a \$1 flow, but multiple edges per currency are allowed



Definitions

- S_{odc}: Total value of transactions (o, d, c) (number of links).
- $S_{od} \equiv \sum_c S_{odc}$: total value of (directed) transactions for country pair (o, d)
- $S_c \equiv \sum_{o,d} S_{odc}$: total value of global transactions using currency c.
- $S \equiv \sum_c S_c = \sum_{o,d,c} S_{odc}$: value of global transactions.
- $D_{ic} \equiv \sum_j \left(S_{ijc} + S_{jic}\right)$: the *degree* of node (i, c), defined as the number of links pointing into and out of node (i, c).

Setup: Growth Process

In each period one new transaction (edge) is created

- Time can then be denoted with S instead of t

The new transaction in period S:

- With probability $1 - \pi(S)$ in an existing currency with (o, d, c) selected by the score function:

$$p_{odc} = \frac{f_{odc}}{\sum_{o',d',c'} f_{o'd'c'}}$$

- With probability $\pi(S)$ a new currency is $\hat{c}$ created for a transaction $(o, d, \hat{c})$ selected with probability

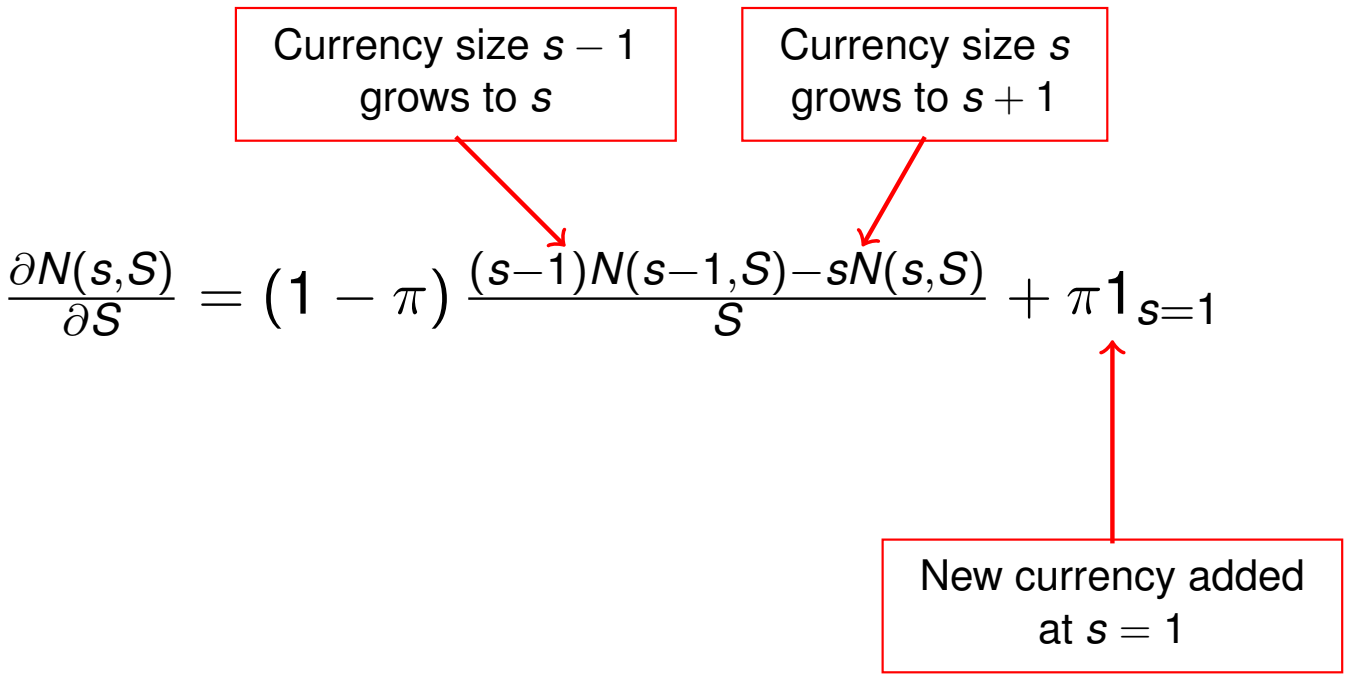
$$\frac{\sum_{c' \neq \hat{c}} f_{odc'}}{\sum_{o',d',c' \neq \hat{c}} f_{o'd'c'}}$$

Functional form for score function:

$$f_{odc} = D_{oc} + D_{dc}$$

Evolution of International Currencies

$N(s, S)$: The number of currencies of size s when the network is of size S follows



Rank-Size Distribution

Proposition 1: Assume constant entry probability $\pi(S) = \pi$. Then market shares of large currencies (large s) follow a power law $\frac{N(s,S)}{S} = s^{1-\zeta}$, with $\zeta = \frac{1}{1-\pi}$

Corollary: Currencies' size-rank relationship is given by the counter-pdf $\bar{F}(s, S) = s^{-\zeta}$

Implication

- With currency creation $\pi > 0$ and $\zeta > 1$
- As entry goes to zero $\pi \to 0$ Zipf's law holds at the limit: $\zeta \to 1$
- **Cannot get hyper-Zipf distribution** $\zeta < 1$

Simulations

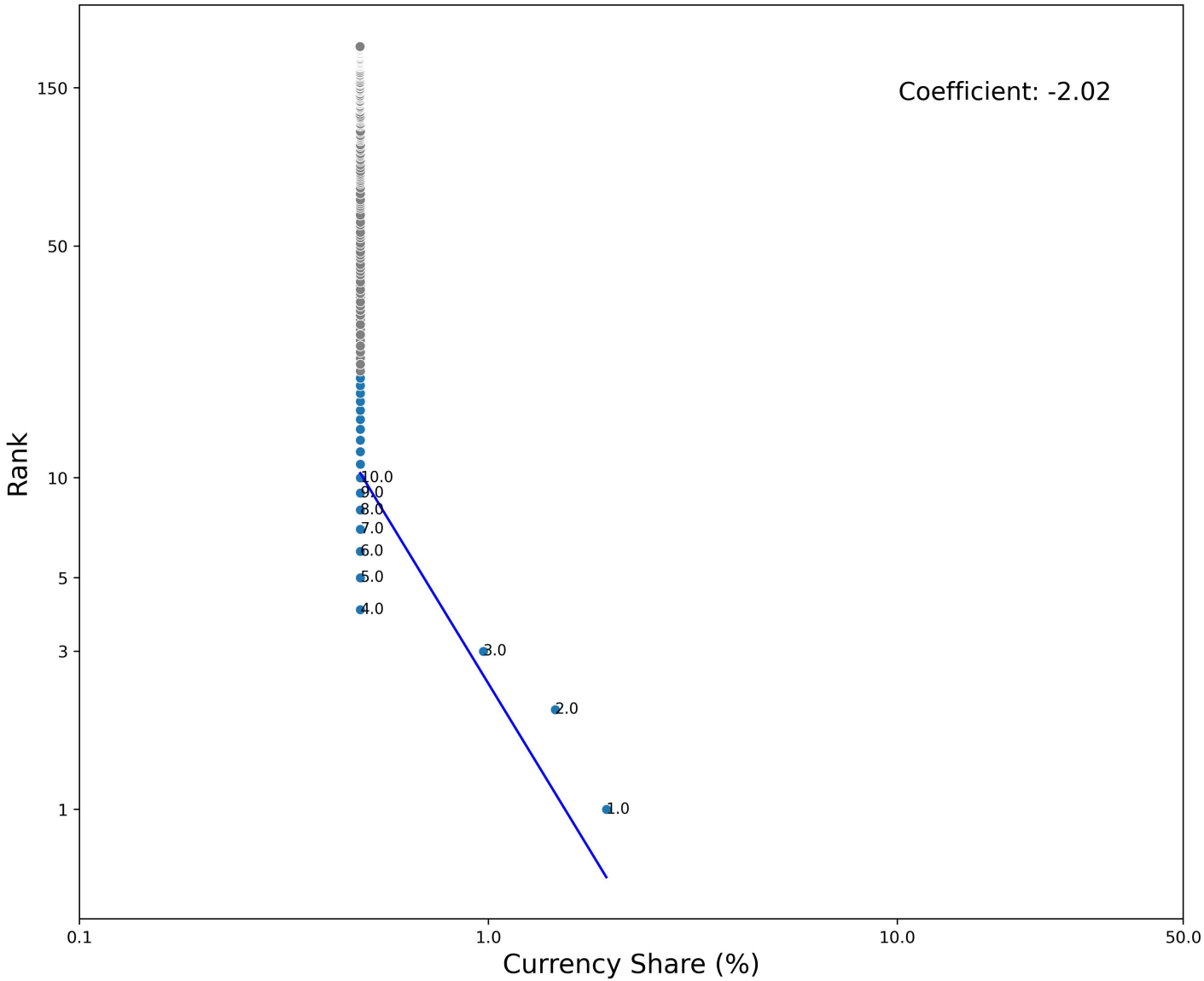
Simulate the model 100K periods. Average over 100 runs.

Zipf's law holds

But top 3 are typically outliers

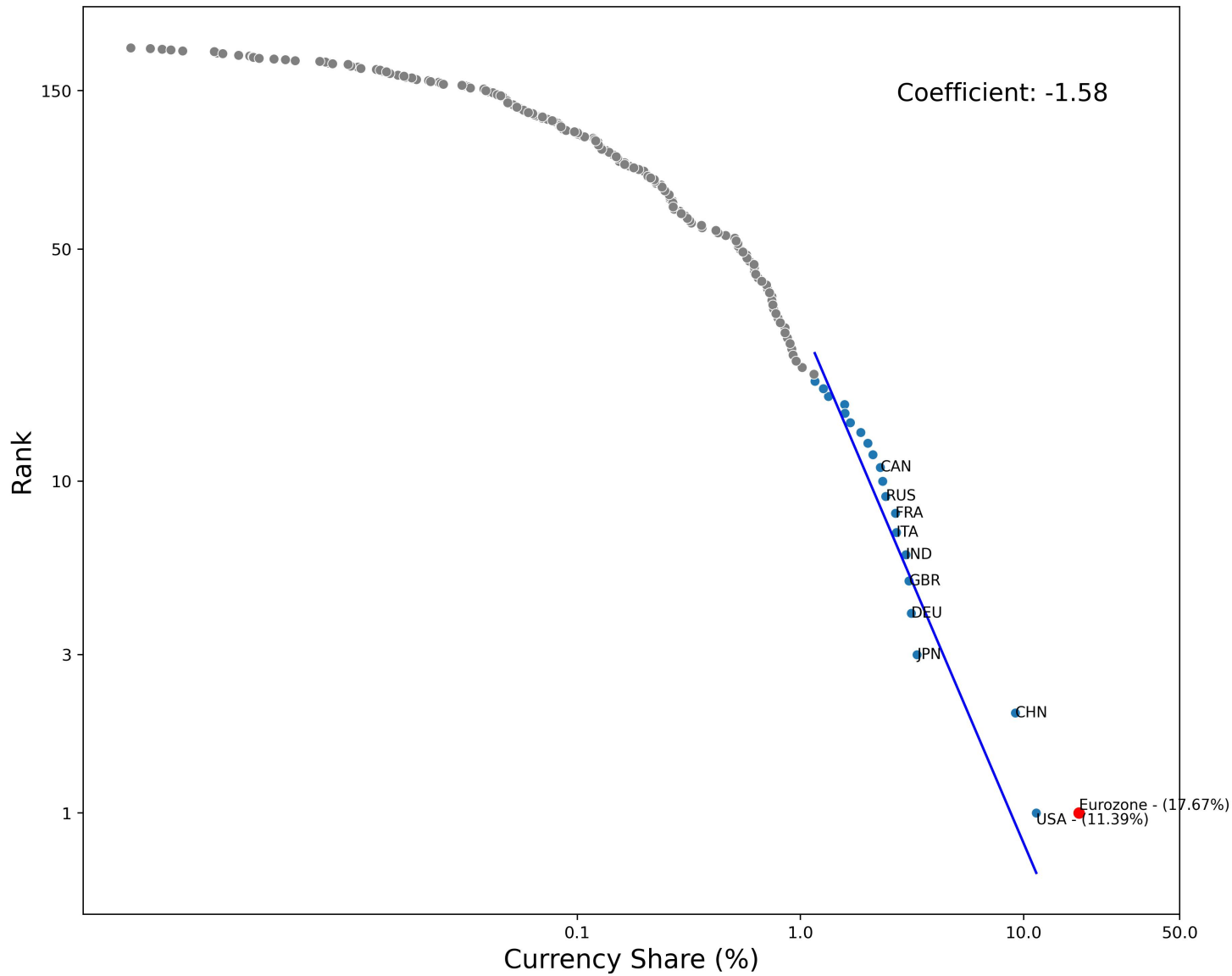
Simulation: Random Growth

Random growth only. Countries same size.

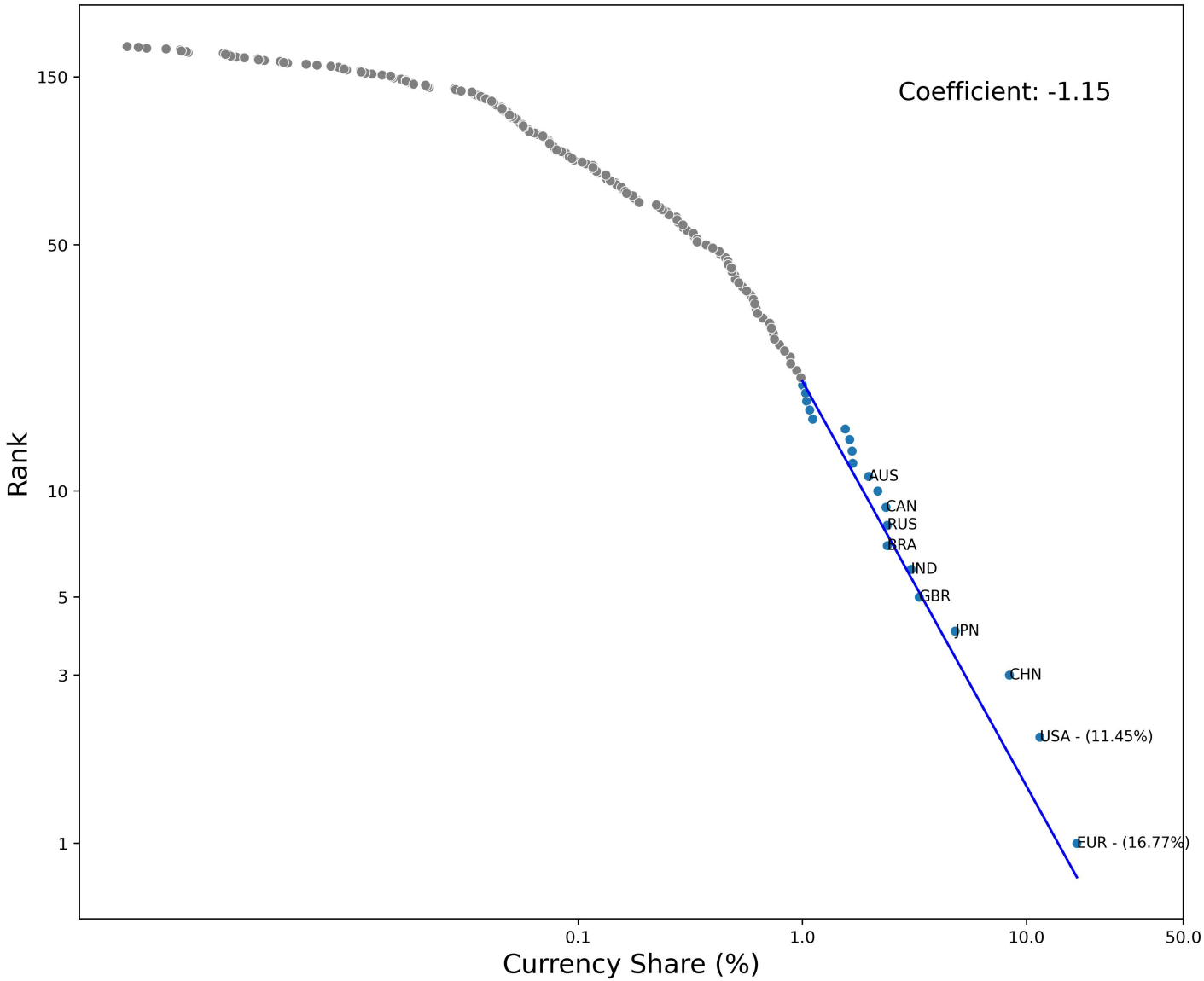


Simulation: Country Size Distribution

Country sizes calibrated. Initially own currency domestically.

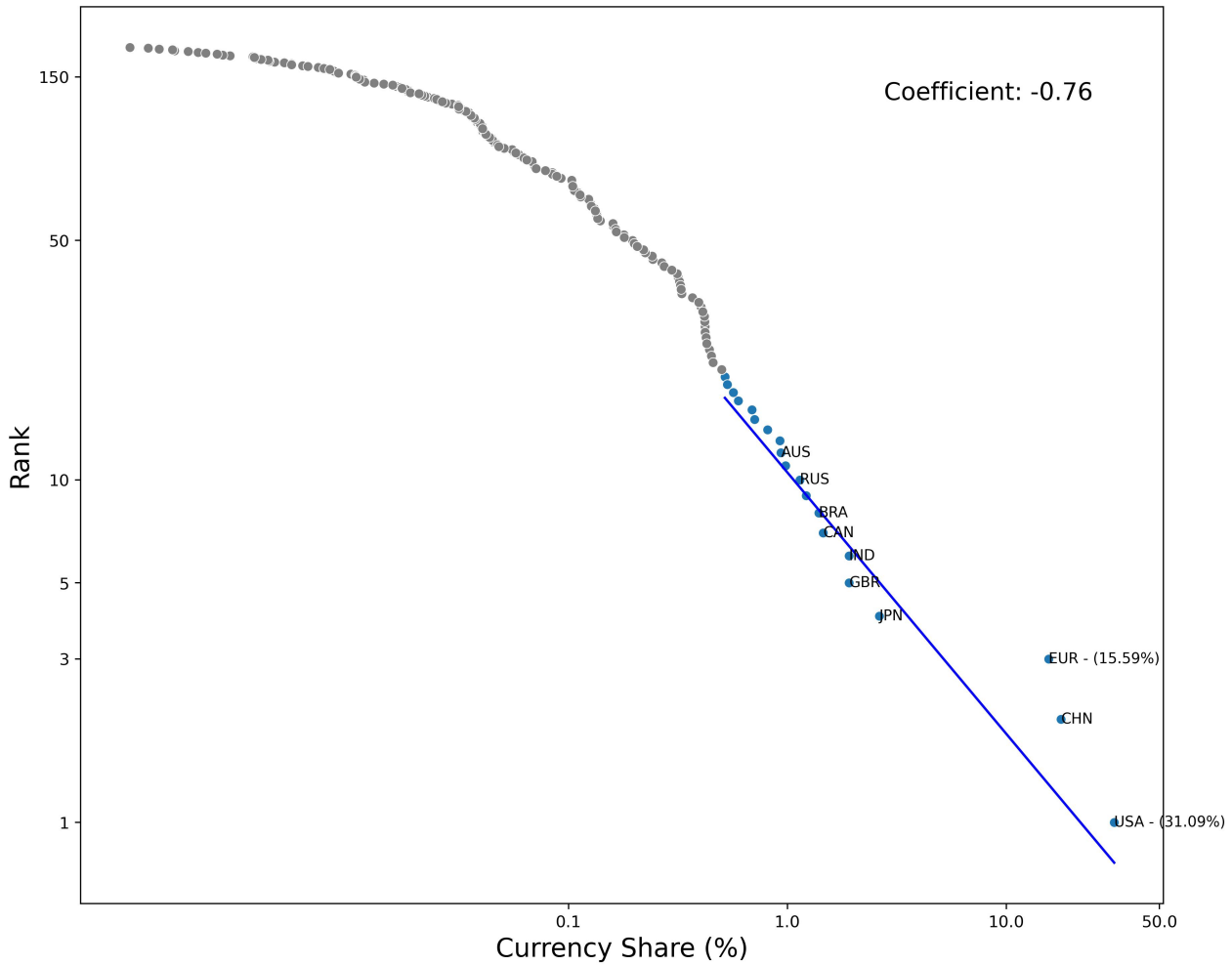


Simulation: Consolidating the Eurozone



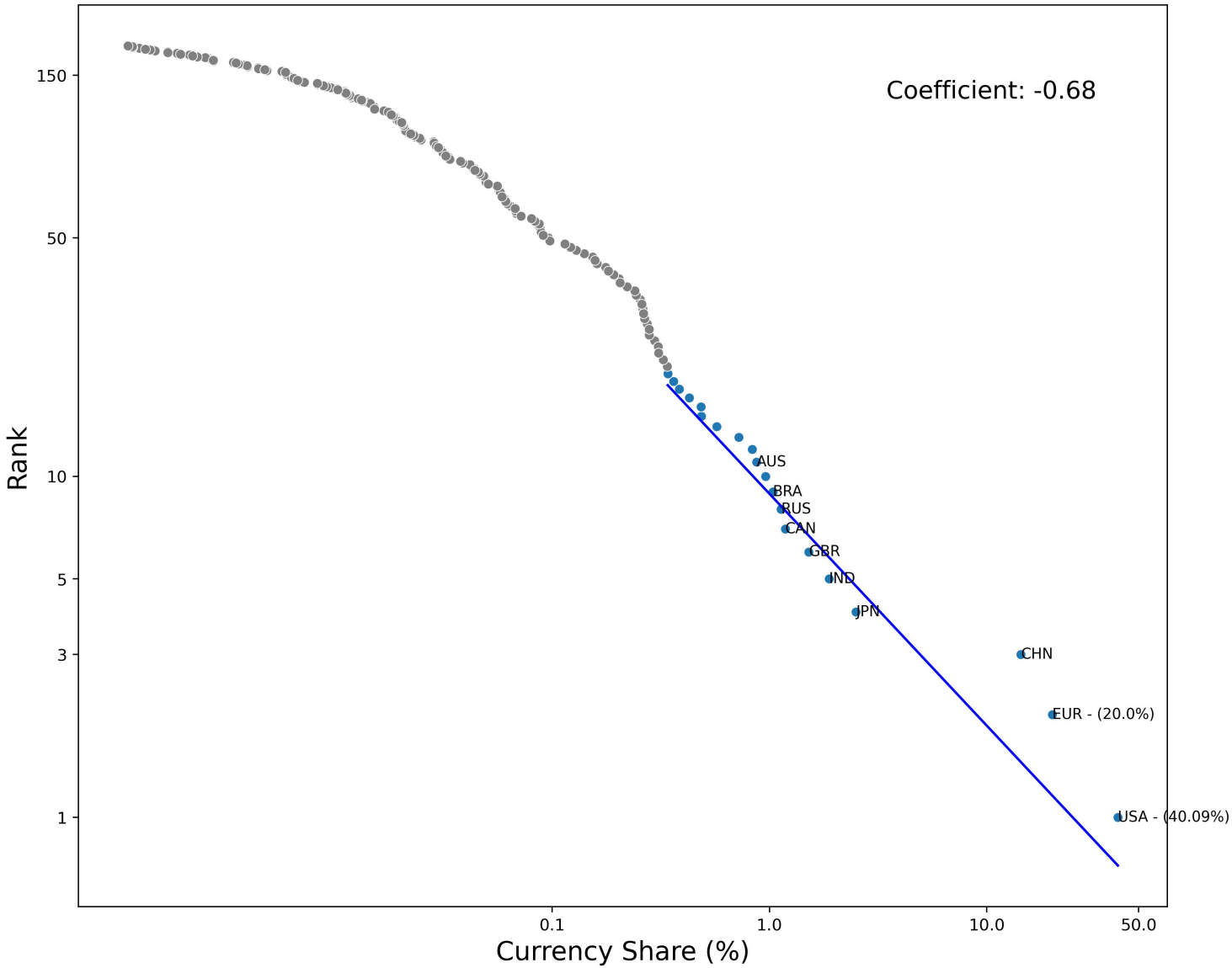
Simulation: Preference for Domestic Currency

Calibrated probabilities of choosing O or D currencies



Simulation: USD and EUR Preference

Calibrated USD when US is trading partner and same for EUR



Contribution

Power law for international currencies: **hyper-Zipf** distribution

US unique: nearly all its in- and out-flows in own currency

This then spreads to greater global dollar dominance

- currency use for other country pairs affected by local popularity of a global currency in the two partners' transactions

Random growth model with local preferential attachment can explain Zipf's law; additional elements to explain hyper-zipf

Appendix

MT103

MT103 messages: communications about payments where either the ordering customer (sender) or beneficiary (recipient) is a non-bank entity.

- Commercial trade transactions
- B2B
- Remuneration payments (intl. payrolls and pensions)
- Interpersonal transfers (remittances)

Far comprehensive than merely “trade”, but excludes purely financial transactions.

Two messaging methods:

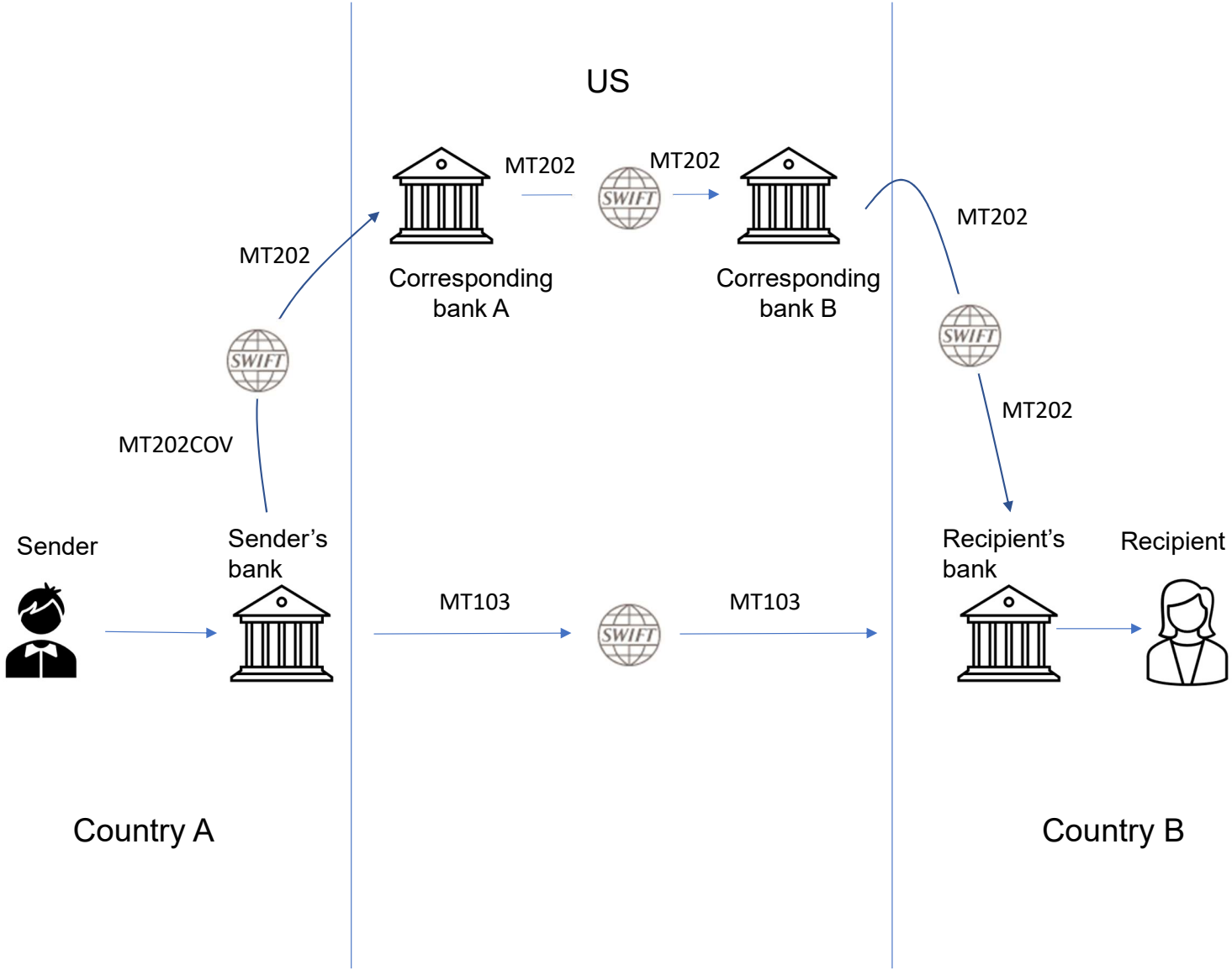
- Cover payments
- Serial payments

return

Cover Payment

Consider a US$ transfer from country A to country B, cleared through the US, via **cover payment**

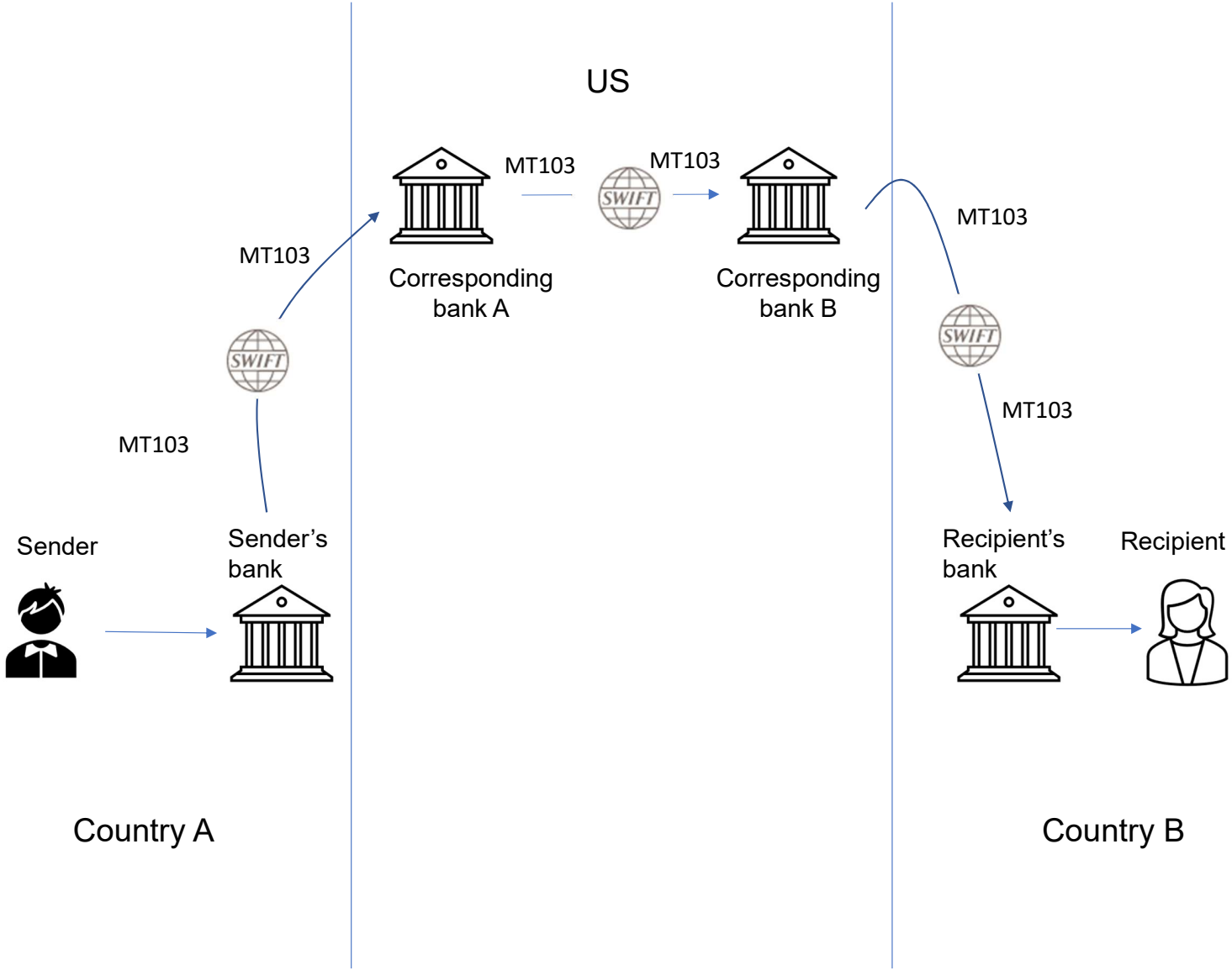
return



Serial Payment

Consider a US$ transfer from country A to country B, cleared through the US, via **serial payment**

return

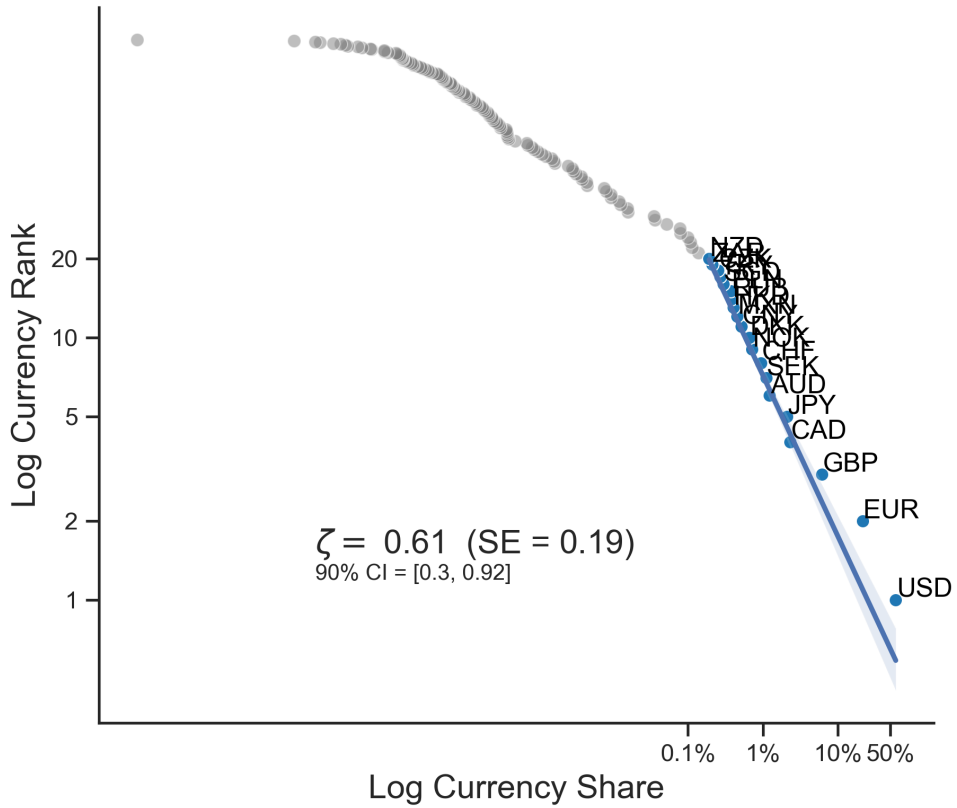


Zipf's Law Everywhere

- City size ($\zeta = 1$): Gabaix (1999), Gabaix & Ioannides (2004)
- Firm size ($\zeta = 1$): Axtell (2011), Fujiwara (2004), Luttmer (2007)
- Income and Wealth ($\zeta = [1.5, 3]$): Champernowne (1953), Klass et al. (2006), Nirei & Souma (2007), Gabaix et al (2016)
- Stock market: returns ($\zeta = 3$) Gopikrishnan et al. (1999), Trading volume ($\zeta = 1.5$): Gopikrishnan et al. (2000)
- Trade: Large gravity literature
- Linguistics: Zipf's law for words (1932, 1935, 1949), Mandelbrot
- Biology: Number of species within genera ($\zeta = 1.4$): Yule and Willis (1922)
- Number of links for websites ($\zeta = 1$): Barabasi and Albert (1999)
- Citations ($\zeta = 1$) de Solla Price (1965) Return

Zipf's Law for International Currencies

All Currencies

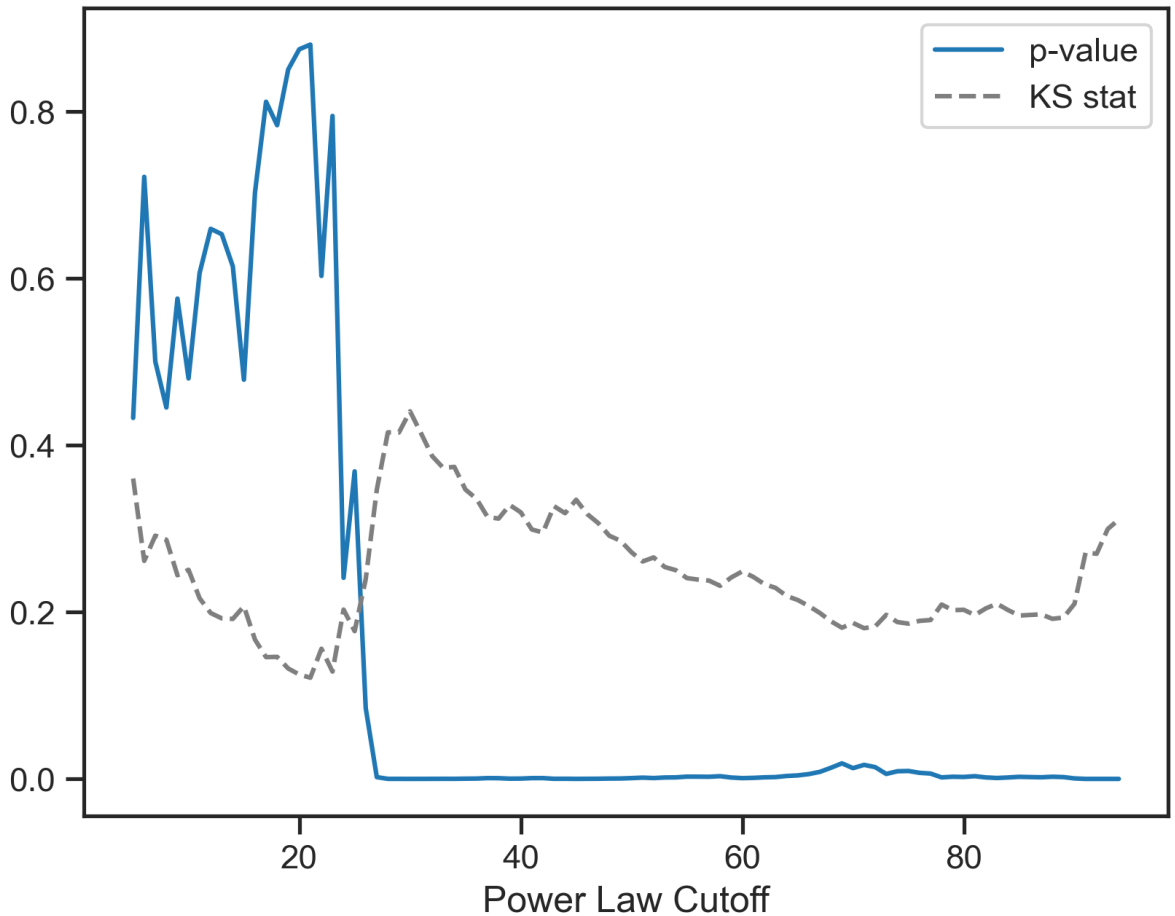


Return

All intl. transactions ex. intra-EZ and intra-China. Top-20 currencies. Line represents OLS log-log regression with Gabiax-Ibragimov bias correction; 95% confidence interval is shaded. Slope of the regression and small-sample-adjusted standard errors reported. Sources: SWIFT and the authors

Zipf's Law for International Currencies

Kolmogorov-Smirnov Test

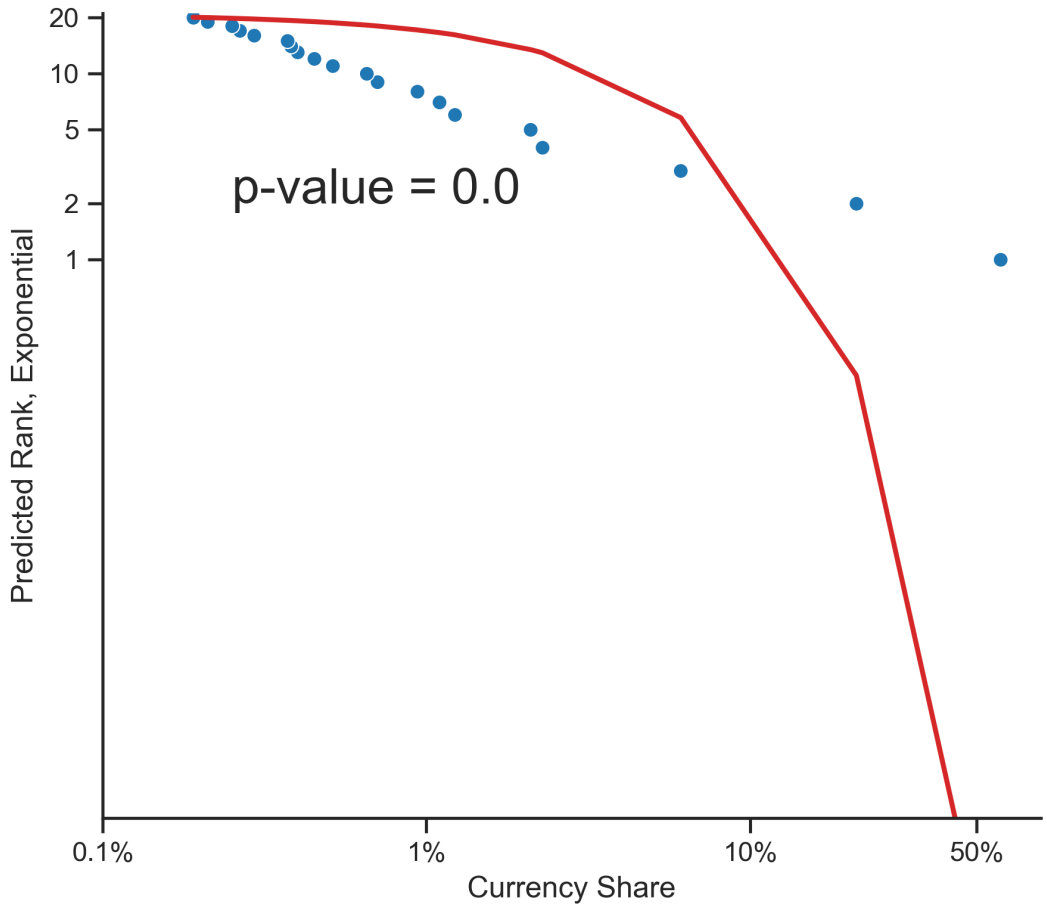


Return

P-stat for Kolmogorov-Smirnov test with varying sample size of top-X currencies. Null hypothesis: linear relationship between log(size) and log(rank). All intl. transactions ex. intra-EZ and intra-China. Sources: SWIFT and the authors

Zipf's Law for International Currencies

Fitting Exponential Distribution

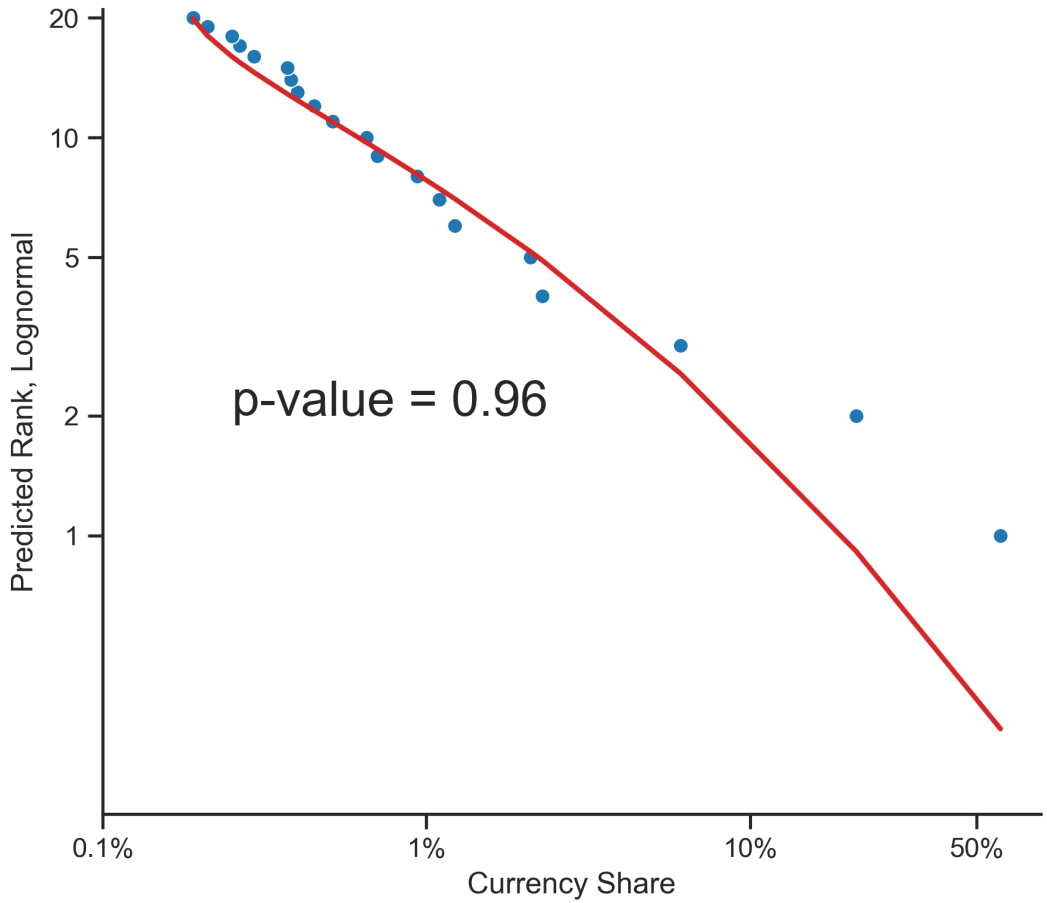


Return

All intl. transactions ex. intra-EZ and intra-China. Sources: SWIFT and the authors

Zipf's Law for International Currencies

Fitting Log-Normal Distribution

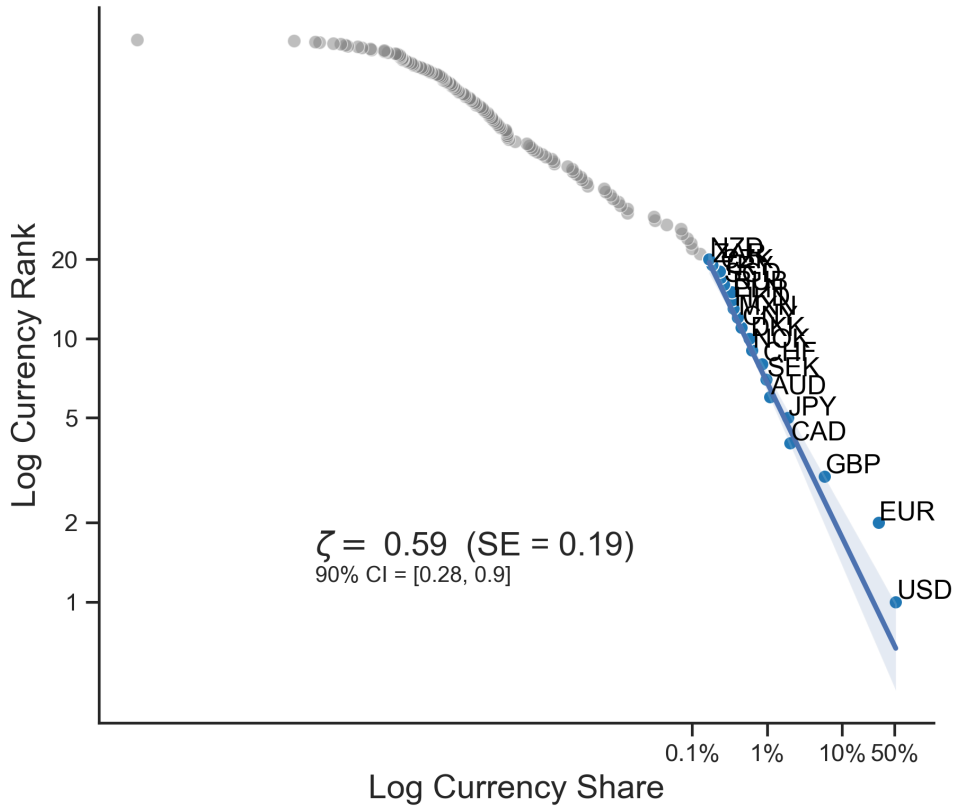


Return

All intl. transactions ex. intra-EZ and intra-China. Sources: SWIFT and the authors

Zipf's Law for International Currencies

Including Intra-Eurozone and intra-China transactions

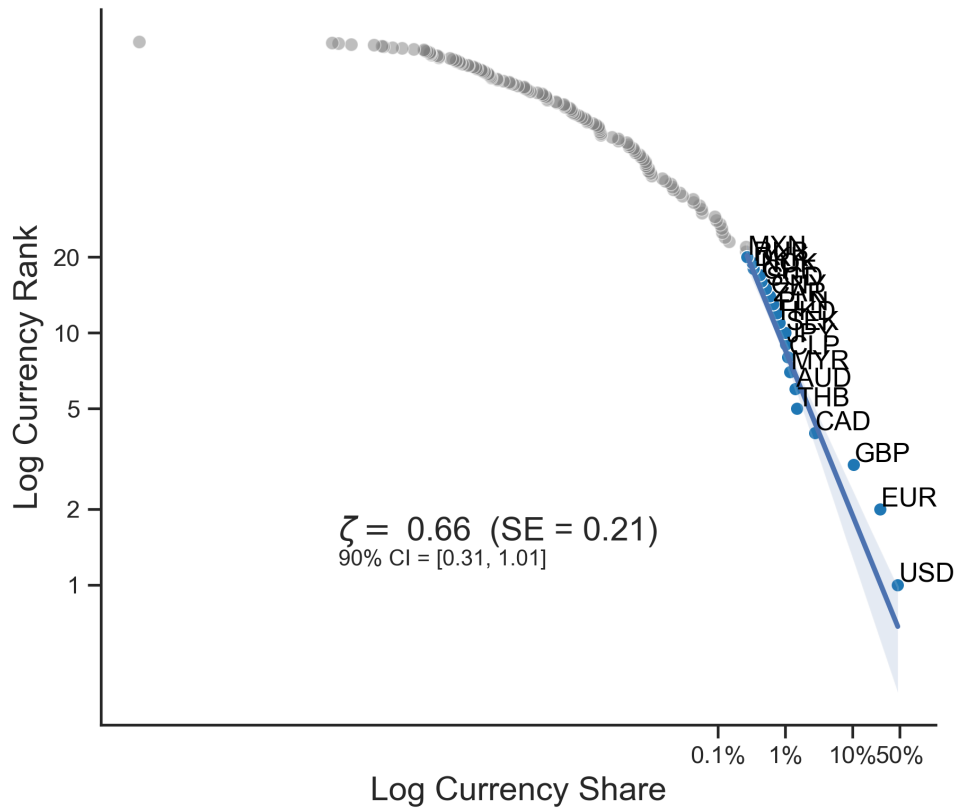


Return

All intl. transactions including intra-EZ and intra-China. Top-20 currencies. Line represents OLS log-log regression with Gabiax-Ibragimov bias correction; 95% confidence interval is shaded. Slope of the regression and small-sample-adjusted standard errors reported. Sources: SWIFT and the authors.

Zipf's Law for International Currencies

Including Domestic Transactions

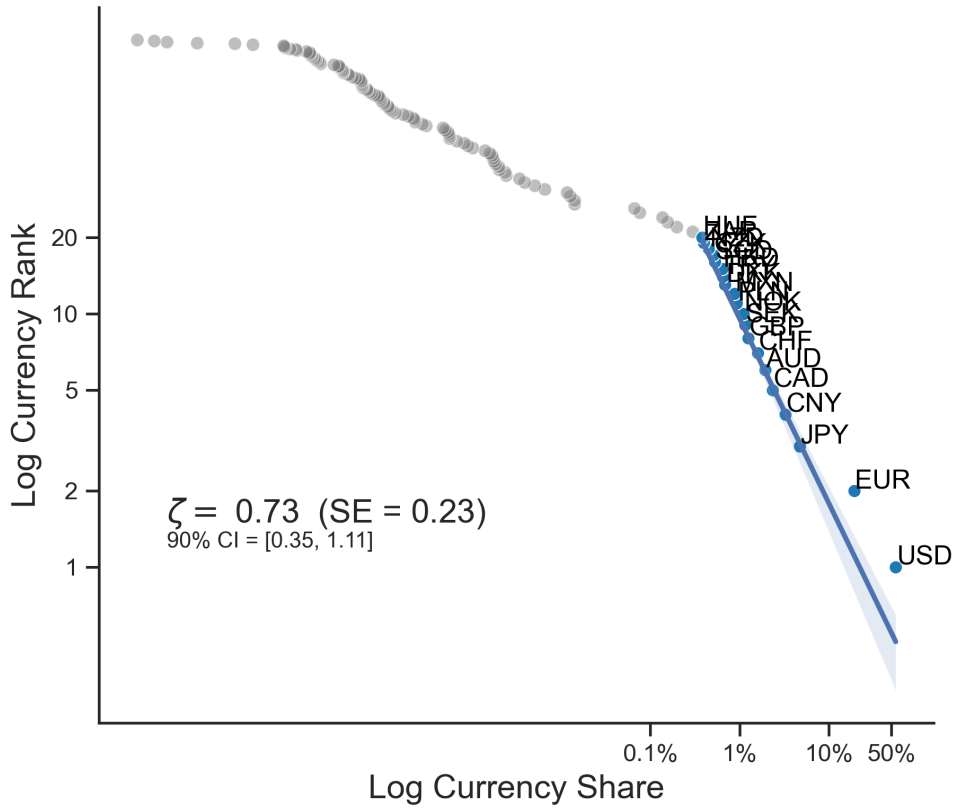


Return

All intl. transactions, including domestic. Top-20 currencies. Line represents OLS log-log regression with Gabiax-Ibragimov bias correction; 95% confidence interval is shaded. Slope of the regression and small-sample-adjusted standard errors reported. Sources: SWIFT and the authors.

Zipf's Law for Vehicle Currencies

All Currencies

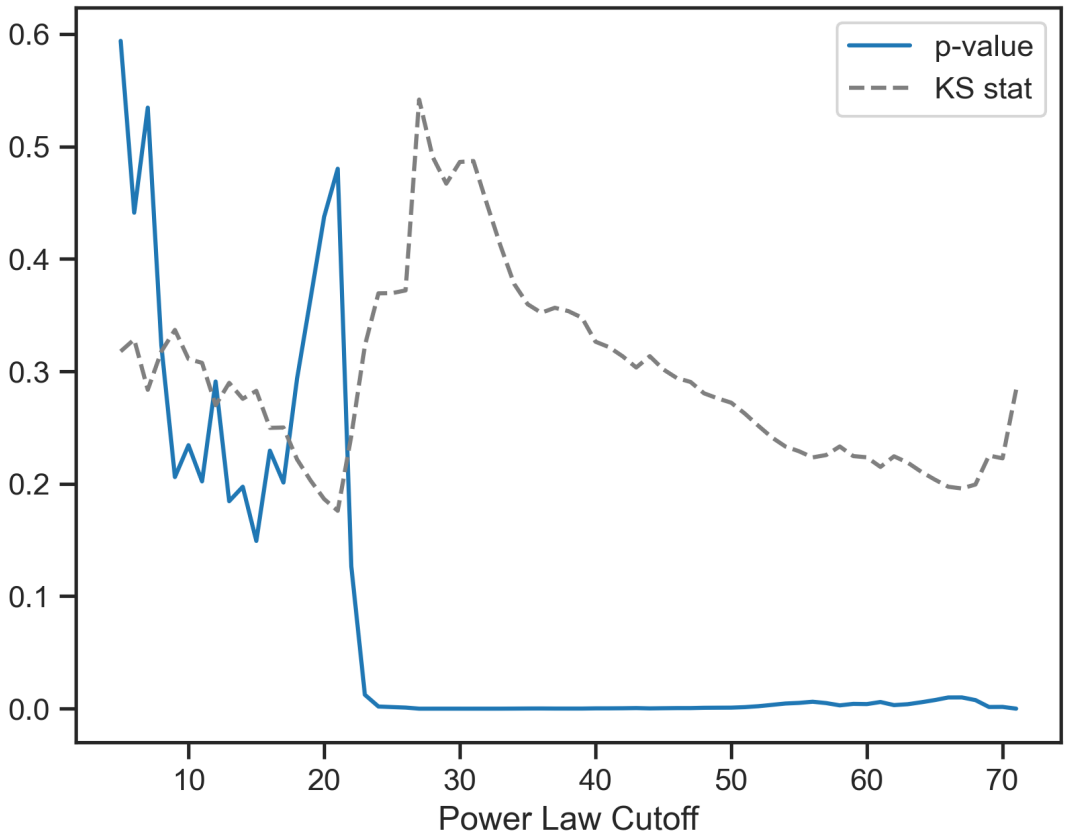


Return

All transactions where currency isn't legal tender in origin or destination country. Ex. intra-EZ and intra-China. 95% confidence interval of OLS log-log regression are shaded. Number of "top" currencies chosen by minimizing the Kolmogorov-Smirnov statistic. Sources: SWIFT and the authors

Zipf's Law for Vehicle Currencies

Kolmogorov-Smirnov Test



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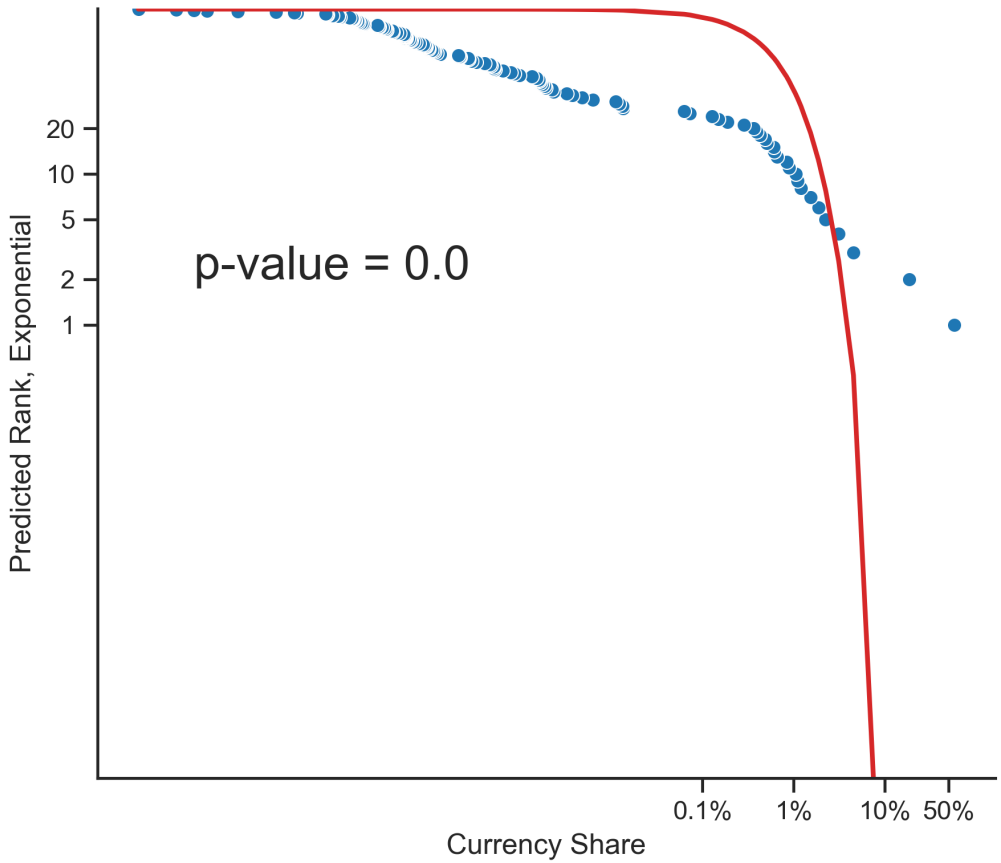
P-stat for KolmogorovâSmirnov test with varying sample size of top-X currencies. Null hypothesis: linear relationship between log(size) and log(rank). All transactions where currency isn't legal tender in origin or destination country. Ex. intra-EZ and intra-China. Sources: SWIFT and the authors

Zipf's Law for Vehicle Currencies

Fitting Exponential Distribution

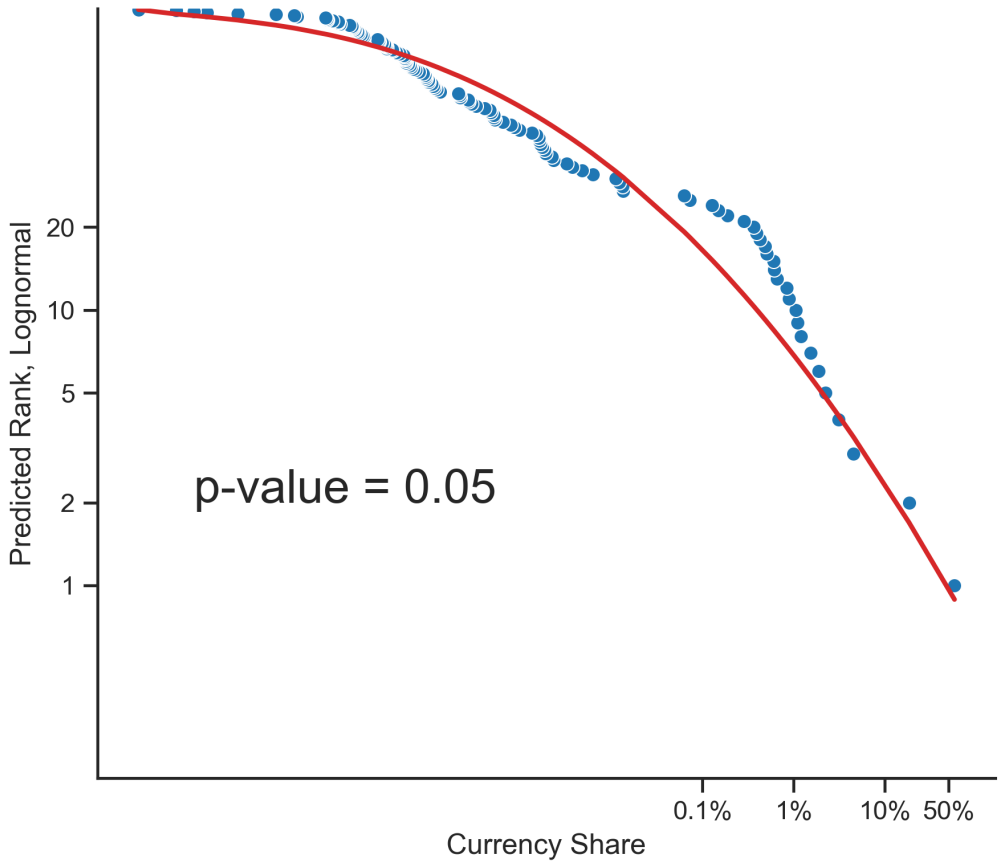


Return

All transactions where currency isn't legal tender in origin or destination country. Ex. intra-EZ and intra-China.
Sources: SWIFT and the authors

Zipf's Law for Vehicle Currencies

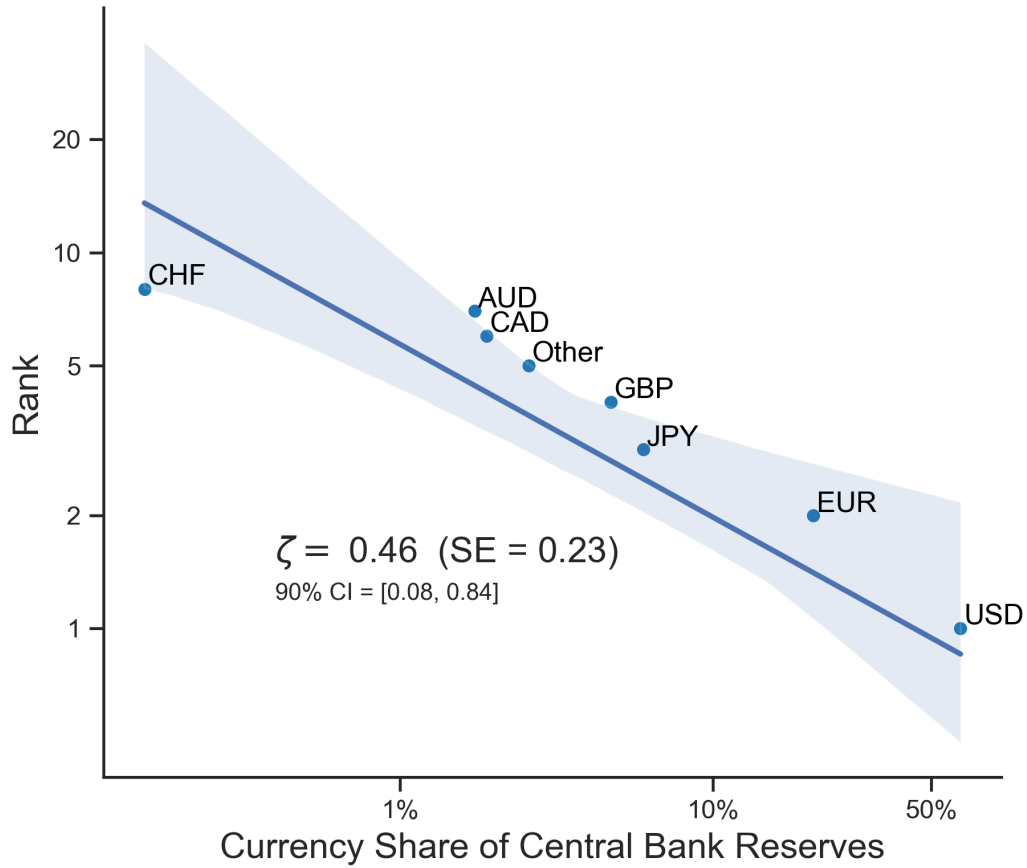
Fitting Log-Normal Distribution



Return

All transactions where currency isn't legal tender in origin or destination country. Ex. intra-EZ and intra-China.
Sources: SWIFT and the authors

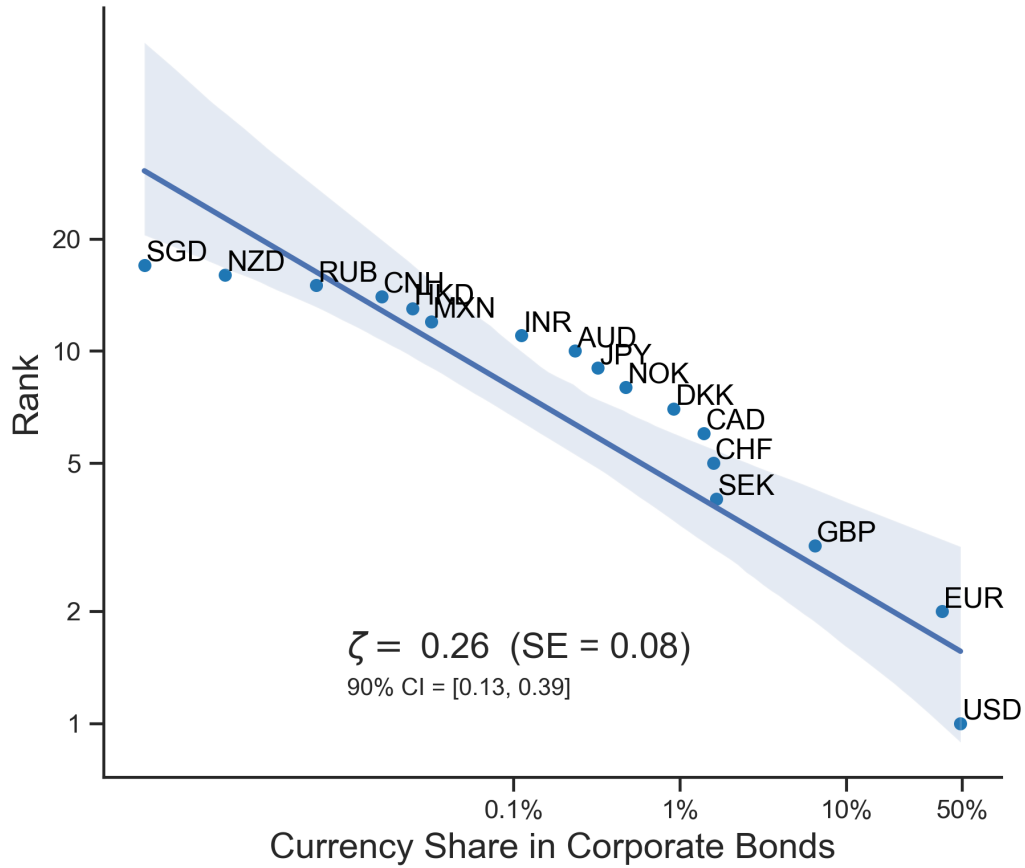
Zipf's Law for Central Bank Reserves



Return

Line represents OLS log-log regression with Gabiax-Ibragimov bias correction; 95% confidence interval is shaded. Slope of the regression and small-sample-adjusted standard errors reported. KS test rejects a linear relationship at the 99% confidence level. Sources: IMF COEFER and the authors

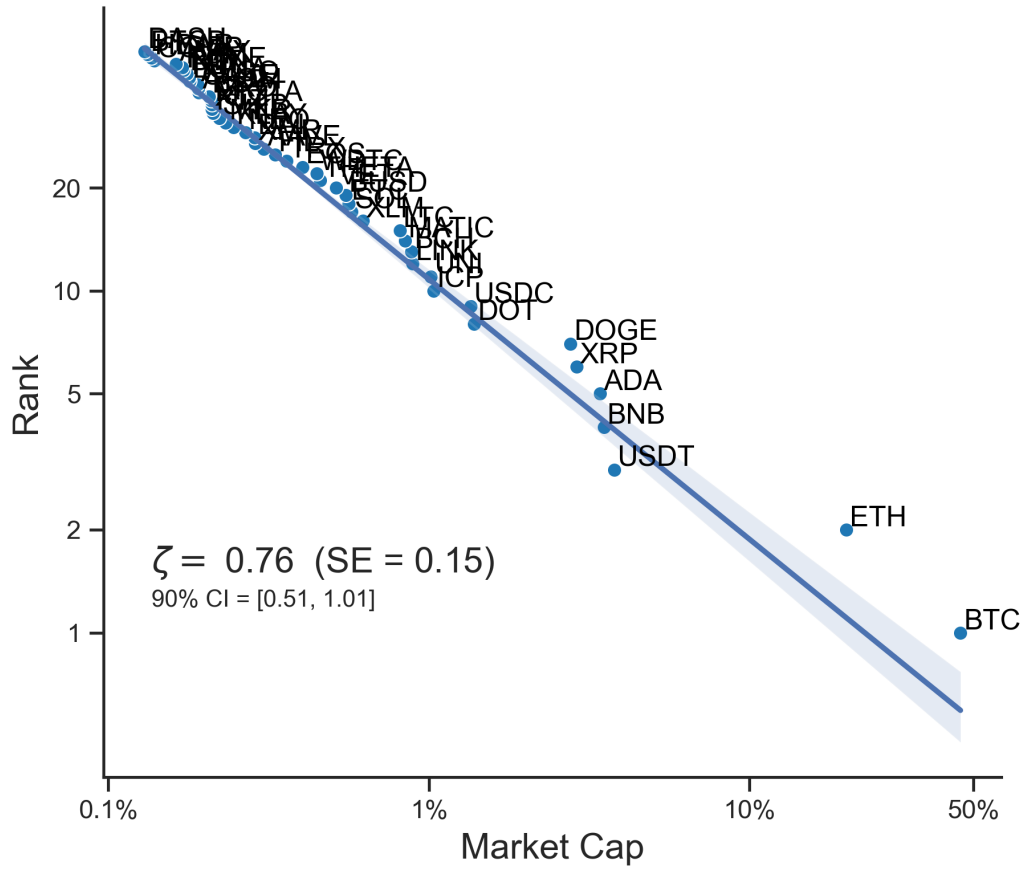
Zipf's Law for Corporate Bond Denomination



Return

Line represents OLS log-log regression with Gabiax-Ibragimov bias correction; 95% confidence interval is shaded. Slope of the regression and small-sample-adjusted standard errors reported. KS test rejects a linear relationship at the 99% confidence level. Sources: Maggiori, Neiman, and Schreger (2020) and the authors

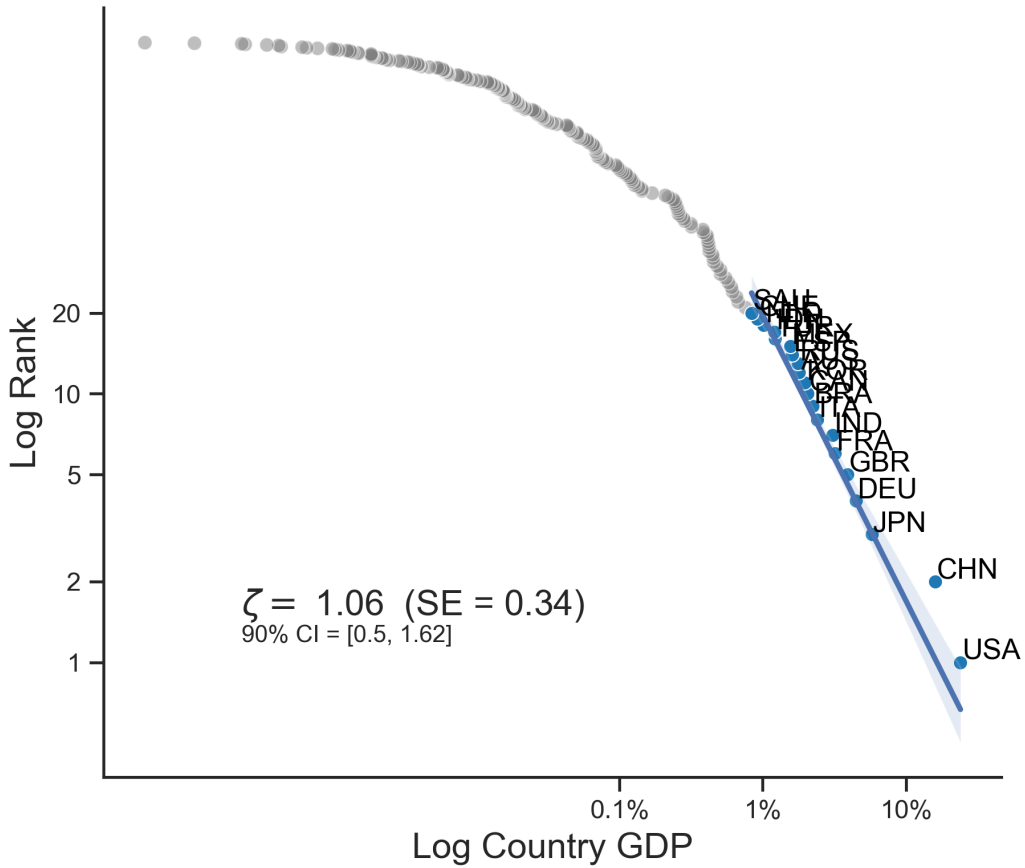
Zipf's Law for Cryptocurrencies



Return

Line represents OLS log-log regression with Gabiax-Ibragimov bias correction; 95% confidence interval is shaded. Slope of the regression and small-sample-adjusted standard errors reported. Sources: coinmarketcap.com and the authors

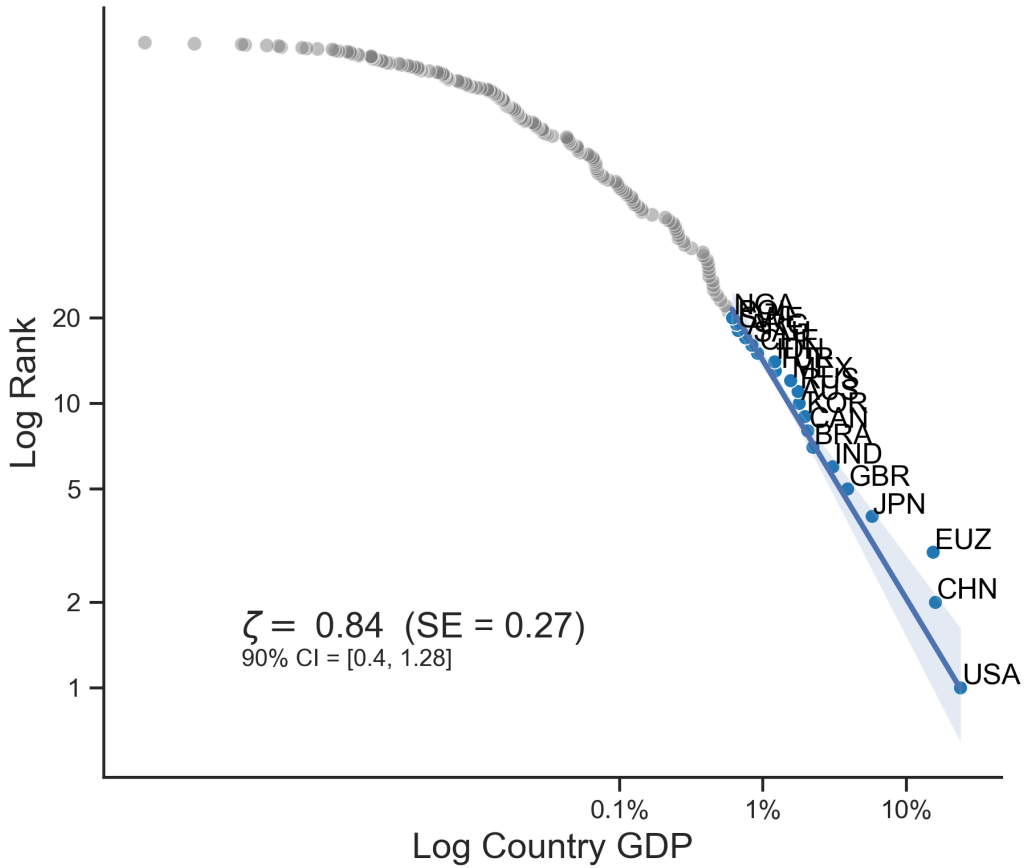
Zipf's Law for GDP



Return

Nominal GDP in US dollars. Line represents OLS log-log regression with Gabiax-Ibragimov bias correction; 95% confidence interval is shaded. Slope of the regression and small-sample-adjusted standard errors reported. Sources: World Bank World Development Indicators and the authors.

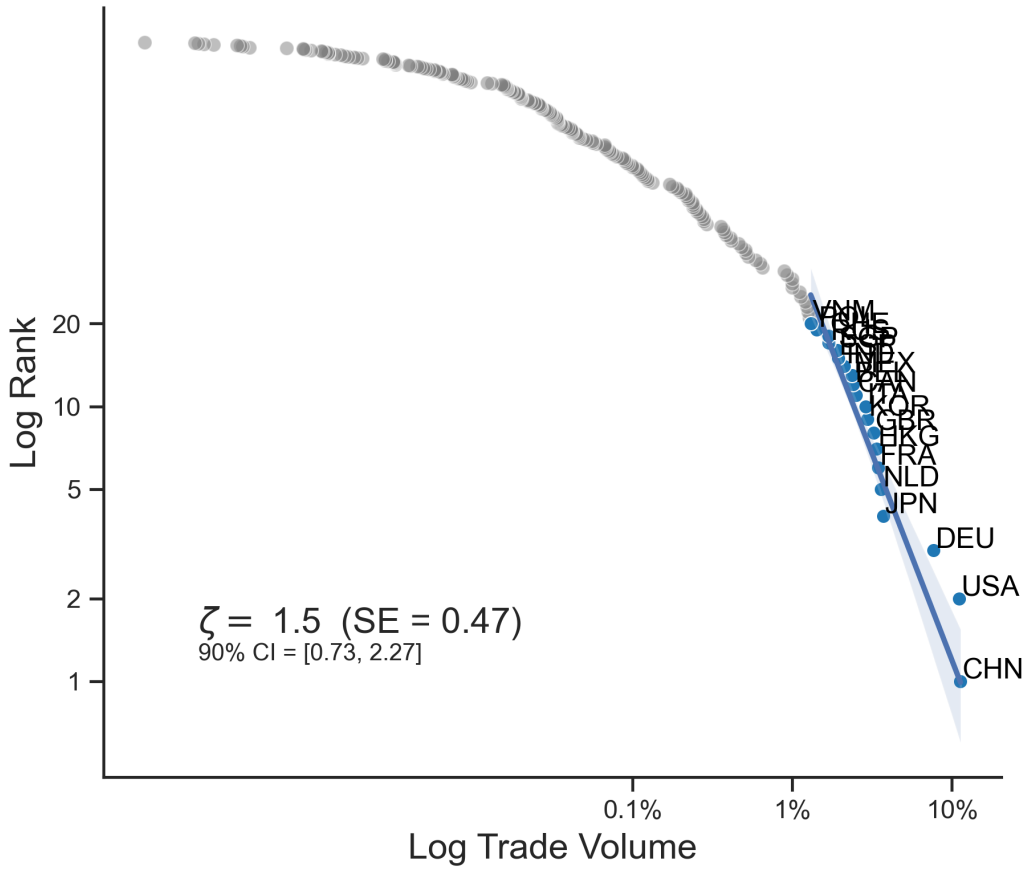
Zipf's Law for GDP (Unified EuroArea)



Return

Nominal GDP in US dollars. Line represents OLS log-log regression with Gabiax-Ibragimov bias correction; 95% confidence interval is shaded. Slope of the regression and small-sample-adjusted standard errors reported. Sources: Penn World Tables and the authors.

Zipf's Law for International Trade

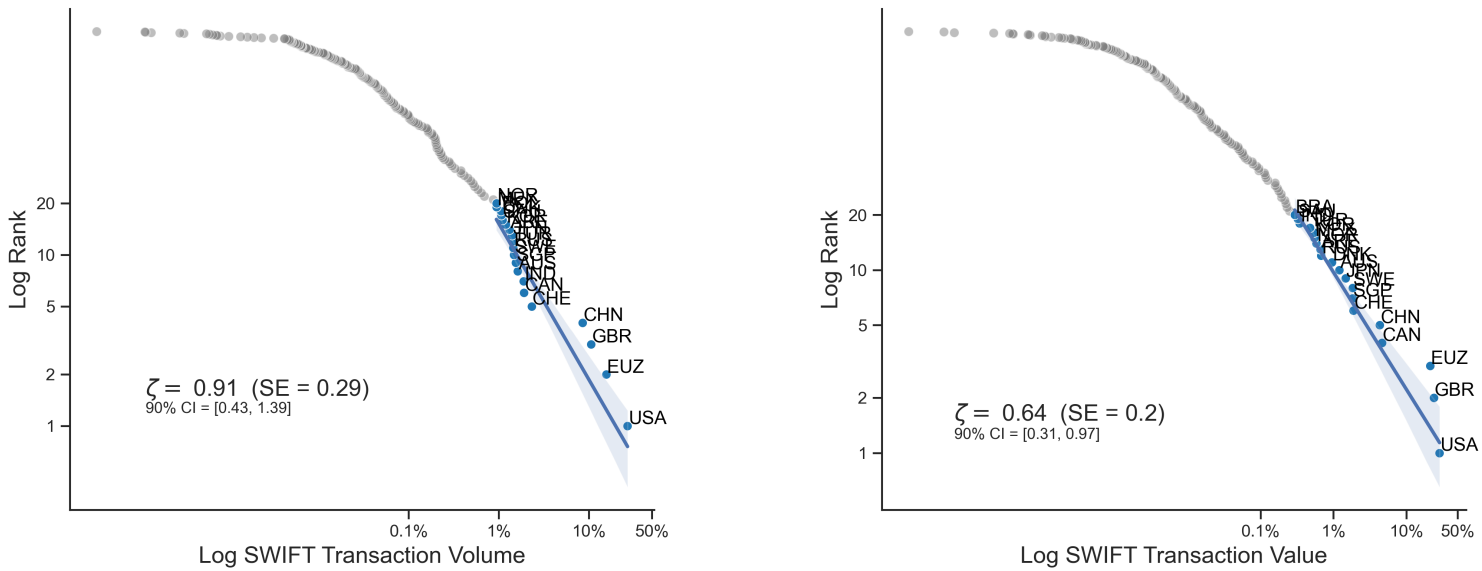


Return

Line represents OLS log-log regression with Gabiax-Ibragimov bias correction; 95% confidence interval is shaded. Slope of the regression and small-sample-adjusted standard errors reported. Sources: UN comtrade and the authors

Zipf's Law for Transactions by Country

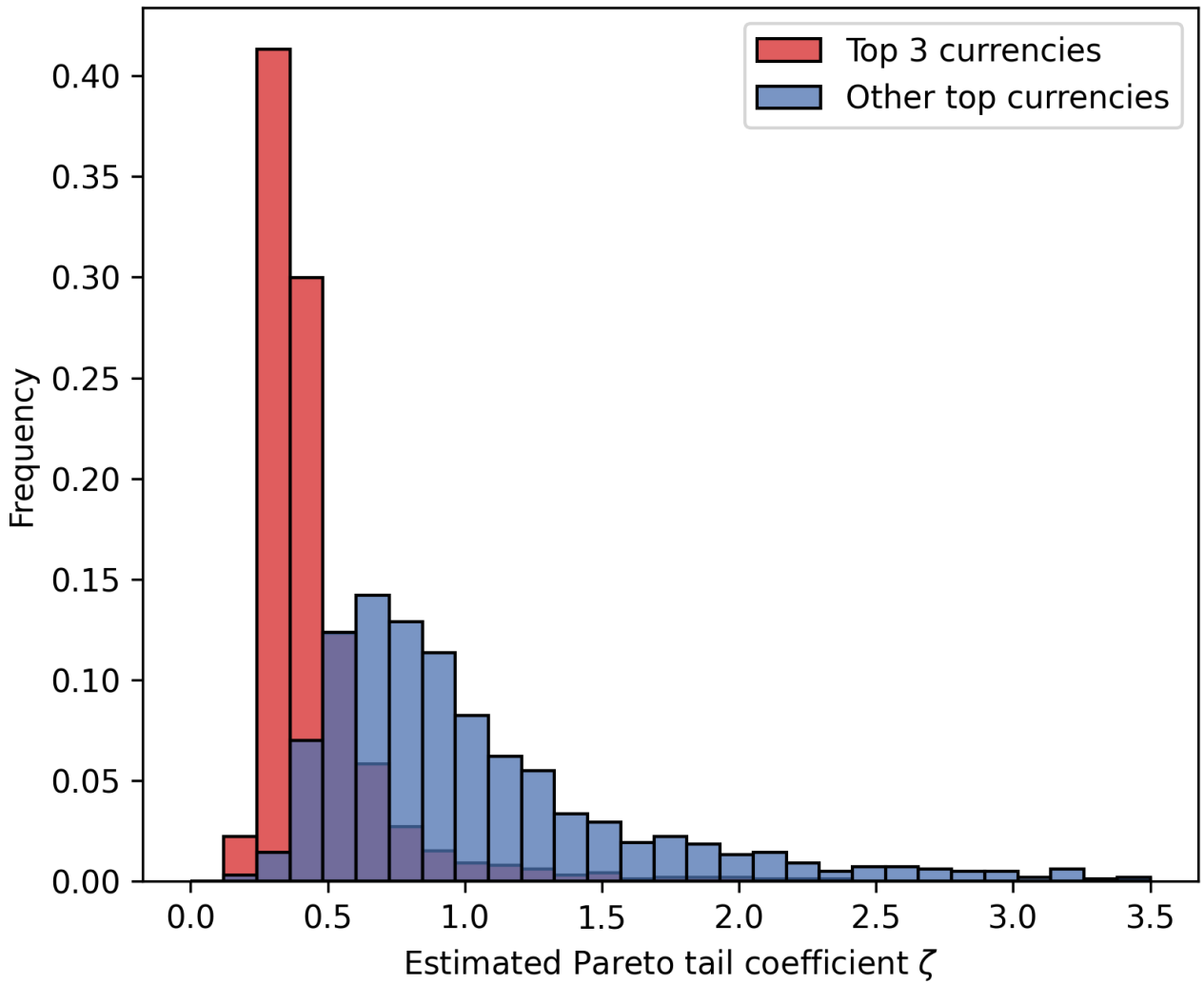
By Volume (left) and Value (Right)



Return

Line represents OLS log-log regression with Gabiax-Ibragimov bias correction; 95% confidence interval is shaded. Slope of the regression and small-sample-adjusted standard errors reported. KS test rejects a linear relationship at the 90% confidence level. Sources: SWIFT and the authors

Histogram of Simulations



Sources: Authors' simulations Return