Fiscal Rules and Market Discipline

Ethan Ilzetzki* [1] and Heidi Christina Thysen[2]

[1]London School of Economics, Centre for Macroeconomics, and CEPR

[2]Department of Economics, Norwegian School of Economics

May 24, 2024

Abstract

Fiscal rules have proliferated as a way to limit public debt. Rules aim to impose fiscal discipline on governments that might be otherwise present-biased. However, lenders also discipline government borrowing through a market mechanism, with excessive debt penalized with higher interest rates. In this paper, we study the interaction between fiscal rules and market discipline in limiting government borrowing. We do so in a sovereign borrowing model with asymmetric information about governments' propensity to over-borrow and default. Governments may signal their fiscal rectitude by showing fiscal restraint and this can lead not only to over-borrowing as in traditional models, but also to under-borrowing, as governments attempt to signal their fiscal responsibility. In addition to their traditional role of restraining present-biased governments, fiscal rules also make signalling more difficult and may have the perverse effect of forcing prudent governments to save even more excessively than otherwise, or alternatively hamper their ability to signal their rectitude entirely. Fiscal rules restrain impatient governments but penalize prudent governments. An optimal fiscal rule balances these trade-offs and will be binding at times, but will never be so tight as to naïvely push governments to "do the right thing".

*This paper was prepared for the 24th Jaques Polak Annual Research Conference. We thank participants in this conference, Prachi Mishra (the editor), the anonymous referees, Olivier Blanchard (discussant), Francesca Caselli, Vitor Gaspar, Evgeniya Kudinova, Gilat Levy, and Ricardo Reis; and seminar participants at the LSE for useful comments and suggestions. Tiago Paul and Vahid Ahmadi provided outstanding research assistance. This project received financial support from UKRI (ERC replacement) grant EP/X025543/1.

1 Introduction

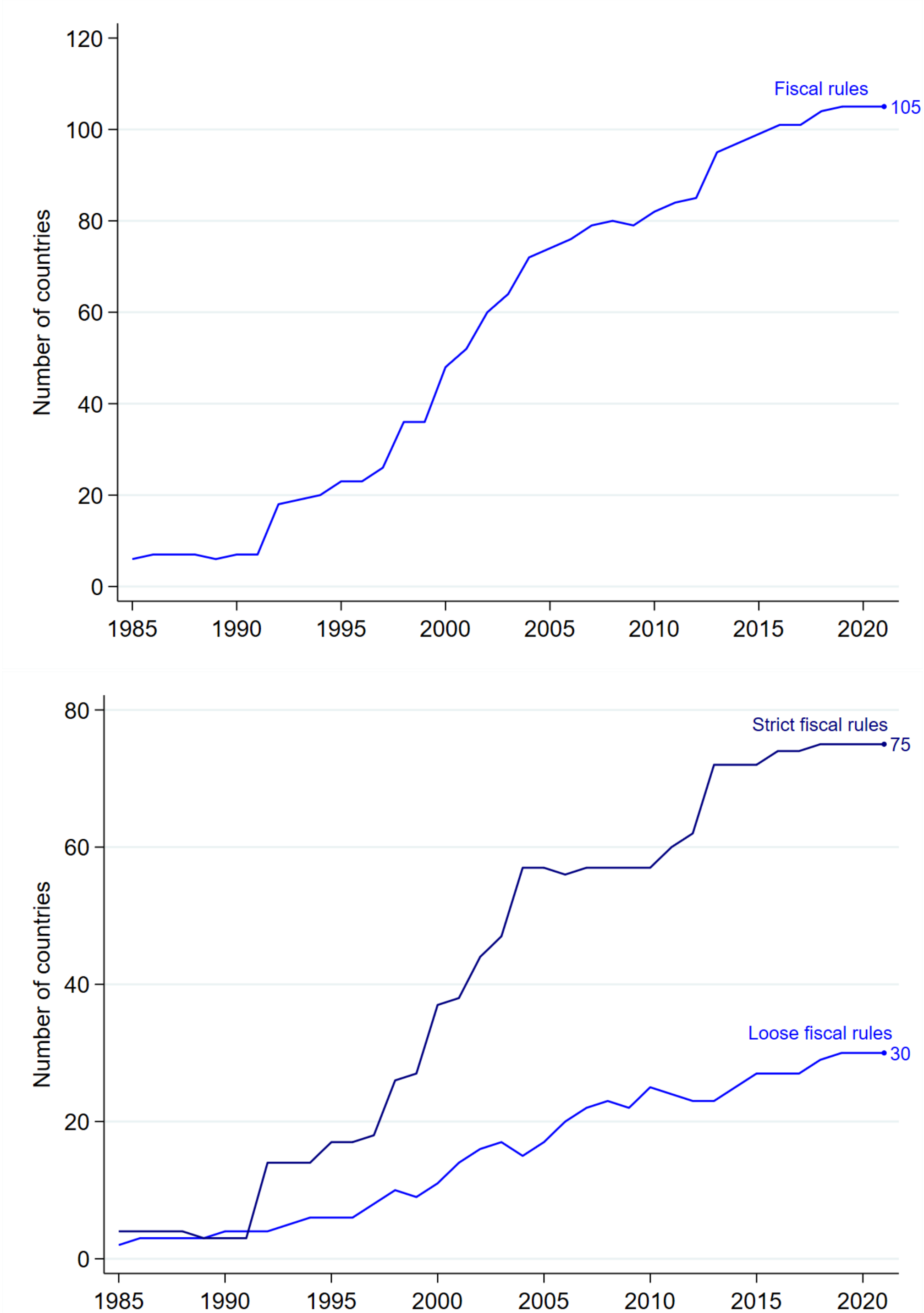
Over the past three decades, governments around the world have adopted fiscal rules to promote fiscal rectitude (Figure 1). These rules vary in their strictness, enforcement mechanisms, and whether they were self-motivated or required by international treaties. Rules stipulate limits on debt, deficits, and government spending. They aim to contain public debt growth and, in some cases, to help promote countercyclical fiscal policies. Perhaps the most prominent rules are those of the European Union and these are binding for its member states under the Maastricht treaty. The EU's rules are still evolving and continue to be a subject of debate among policymakers and economists (Zettelmeyer *et al.*, 2018; Bilbiie *et al.*, 2021; Ilzetzki, 2021; Arnold *et al.*, 2022; Thygesen *et al.*, 2022).

Debates and academic research have focused primarily on the tradeoff between credibility and flexibility in designing fiscal rules (Halac & Yared, 2014; Blanchard *et al.*, 2021; Campante *et al.*, 2021; Barnichon & Mesters, 2022). Stricter and less flexible rules are easier to monitor and may be more credible in ensuring long-run fiscal sustainability. However, they may lead to pro-cyclical fiscal policies with grave social consequences and can exacerbate business cycles. On the other hand, rules allowing more flexibility and "escape clauses" can be designed to permit more countercyclical policies, but they may lack credibility.

This paper investigates a different–and previously unexplored–tradeoff in the design of fiscal rules: the tradeoff between discipline imposed by markets and peers and those imposed by rules. The market mechanism imposes fiscal discipline by increasing interest rates when the government's willingness to repay its debt is in question. The desire to ensure cheaper funding and better market access gives governments an incentive to limit their borrowing. The common rationale for fiscal rules is that governments are more impatient (present-biased) than their citizens due to political economy forces (e.g. government turnover as in Alesina & Tabellini 1990 or legislative bargaining as in Azzimonti *et al.* 2016. Yared 2019 provides a survey of this literature.) These political forces not withstanding, governments may attempt to signal their fiscal rectitude and credibility by running lower deficits (or larger surpluses) to secure lower borrowing rates (Gibert, 2022). This signalling motivation ensures a degree of fiscal discipline even absent fiscal rules. The premise of this paper is that fiscal rules interact with market discipline and signalling motivations in non-trivial ways. Ignoring this interaction may lead to sub-optimally designed fiscal rules.

We develop a model of asymmetric information, where governments differ in their degree of present bias. Policymakers know their own type, but markets cannot observe this directly: a reasonable assumption, particularly when the policymaker is newly elected or facing new circumstances. The market must infer policymakers' types from their actions. Policymakers may then restrain their borrowing themselves to signal to the market that they are fiscally responsible. We

Figure 1: Fiscal Rules Have Proliferated



Note: The figure shows the number of countries with a fiscal rule. The top panel shows all types of fiscal rule. The bottom panel shows separately stricter and looser fiscal rules. Source of data and of rule classification: IMF Fiscal Affairs Division.

show that this may lead policymakers not only to over-borrow at times because of their present bias, but also under-borrow at other times to signal their fiscal responsibility to markets. In fact, under-borrowing may occur even if *all* policymakers are present-biased.

We then consider what happens when a debt limit is imposed on the government. Fiscal rules may affect the behavior of less-responsible governments by restricting their borrowing. In our context, this always benefits citizens because there is no uncertainty or asymmetric information about economic conditions. However, the debt limit negatively impacts more responsible policymakers, even though it doesn't constrain them directly. They must now run even lower deficits to demonstrate their prudence which could lead them to under-borrow even further. Fiscal rules can therefore either improve or harm economic welfare. The analysis in this paper demonstrates that it is never optimal to adopt a naïve fiscal rule that forces an irresponsible government to "do the right thing". The optimal fiscal rule gives irresponsible governments some leeway avoid excessive austerity by more responsible ones, whom–without this analysis–might not seem affected by these rules.

We conclude by discussing how fiscal rules can be reformed based on our findings. Rules that restrict debt or deficits make it harder for markets to asses borrowers' creditworthiness. This is true whether debt is restricted directly or indirectly, such as by forcing governments to stop borrowing when sovereign spreads widen (Hatchondo *et al.*, 2022a,b). Fiscal standards (Blanchard *et al.*, 2021) could cause similar issues: they too restrain extravagant governments and could make it more difficult for prudent policymakers to signal their fiscal responsibility. Instead, we advocate for rules that clarify how sovereign debt will be resolved, whether through legislation or collective action clauses. This approach separates sovereign risk from politicians' present-bias and removes the need to signal fiscal responsibility to markets. Once this clarity is established, deficit limits imposed to address policymakers' present bias not longer have the distorting effects studied here.

This paper was first presented at the 24$^{\text{th}}$ Jacques Polak Annual Research Conference in honor of Ken Rogoff's illustrious research career and it is inspired by several of his contributions. First, the paper is most directly inspired by Ken's work on the optimal degree of commitment to an inflation target (Rogoff, 1985). Ken showed how an inflexible *monetary* policy rule can be sub-optimal. His rationale is the now-standard tradeoff between rules and discretion and presages the recent literature on fiscal rules. Second, it has a similar game theoretic framework as that of Ken's work on political budget cycles (Rogoff, 1990; Rogoff & Sibert, 1988). In Ken's papers, political budget cycles arise because election-year spending may signal a government's competence. In this paper, governments signal their fiscal responsibility by under-borrowing, a problem that is exacerbated by fiscal rules. Third, the paper relates to Ken's extensive body of work on sovereign debt and default (Bulow *et al.*, 1988, 1992; Bulow & Rogoff, 1988, 1989b,a, 1990, 1991; Reinhart

& Rogoff, 2009). Finally, Ken's research on "Debt Intolerance" (Reinhart *et al.*, 2003) shows that governments struggle to improve their credit ratings even decades after a default, highlighting the difficulty competent governments face in signalling that they have turned a page.

This paper also relates to a more recent theoretical literature that investigates the effects of fiscal rules on economic outcomes and proposes optimal fiscal rules. Besley (2007) and Besley & Smart (2007) study the agency problem between voters and politicians. They find that electoral incentives are generally insufficient to prevent rent-seeking politicians from running excessive deficits, laying the groundwork for the question we ask here: can markets discipline politicians to better allign with citizens' preferences. Works by Halac & Yared (2014) and Barnichon & Mesters (2022) study the tradeoff between commitment and flexibility. Asymmetric information is an important friction in Halac & Yared (2014), but the government has superior information about the state of the economy–and thus the need for policy discretion–in these studies. Piguillem & Riboni (2021) study how fiscal rules affect the politics of bargaining over budgets. Dovis & Kirpalani (2020) develop a model of fiscal rules and asymmetric information in a federal setting, but there it is the credibility of the fiscal rule itself that is unknown. Local governments (e.g. EU member states) may breach the externally-imposed fiscal rule to test the central government's (e.g. the EU's) commitment to enforce the rule. There, a fiscal rule may lead counterintuitively to excess borrowing, while we show an opposing force whereby a fiscal rule could lead to excessive austerity. Amador & Phelan (2021) study a dynamic model of reputation that bears some resemblance to the framework we propose here, but they don't allow "responsible" governments to signal their type or consider the effects of fiscal rules.

2 The Model

In this section, we present a straightforward model that conveys the core message of our paper. Later, in Section 4, we enrich the model with additional ingredients. An economy lasts for two periods, $t = 1, 2$. The economy consists of citizens and a policymaker (PM), who chooses public goods in each period, g_1 and g_2, respectively. Citizens and the PM have preferences over public goods according to the utility function

$$U^\theta = u(g_1) + \beta^\theta u(g_2), \tag{1}$$

where u is increasing and concave. There are two types of policymaker, $\theta \in \{P, E\}$, which we will refer to as **P**rudent (she) and **E**xtravagant (he). Nature selects the PM type prior to period 1 and selects a prudent type with probability $\Pr(\theta = P) = \pi$. The prudent and extravagant types differ in two ways. First, a prudent policymaker is more patient than an extravagant one: $\beta^P > \beta^E$. Second, they differ in their probability of defaulting on public debt in the second period. This probability is denoted by δ^θ. The extravagant type is more likely to default: $\delta^P < \delta^E$. In this basic model, default is exogenous. In the extended model of Section 4, the government chooses to default on its debt strategically; there too, prudent policymakers are less likely to default. As we proceed, we will highlight any results that change when default is endogenous.

The government receives an exogenous stream of revenues y_t in period t that can be transformed costlessly, one-to-one, into public goods. The government borrows in period 1 in a competitive lending market with risk-neutral international lenders, who face a gross funding cost of R. The government borrows by issuing bonds at a price q, so that it receives qb units of the consumption good in period 1 when it promises to repay b units of the good in period 2. A government who is known to have a zero probability of default would obtain a bond price of $q = \frac{1}{R}$. We refer to the sovereign spread whenever the bond price deviates from this risk free price. We assume throughout that income growth in the second period (y_2/y_1) is sufficiently high and/or discount factors are sufficiently low so that all government types wish to borrow in equilibrium. We assume a minimal amount of public goods that must be provided in each period: $\underline{g} \geq 0$. This may represent non-discretionary spending that any government must provide, or the minimal amount of public goods that are politically feasible. With this in mind, the government faces the budget constraints

$$g_1 \leq y_1 + qb \tag{2}$$

$$g_2 + b \leq y_2 \tag{3}$$

$$b \geq -\max\{\frac{\underline{g} - y_1}{q}, 0\} = \underline{b}(y_1, q). \tag{4}$$

The first two inequalities are the government's budget constraints in the two periods. The last constraint gives the minimal amount of borrowing required to provide the minimum mandatory amount of public goods $\underline{g}$. Further, the government may face a fiscal rule in the form of a debt limit so that $b \leq \bar{b}$

A type θ PM chooses borrowing b^θ to maximize

$$U^\theta(b^\theta, q) = u(y_1 + qb^\theta) + \beta^\theta[(1 - \delta^\theta)u(y_2 - b^\theta) + \delta^\theta u(y_2)],$$

subject to the minimal borrowing constraint and the debt limit given above. As we will see, the bond price q is itself affected by the government's borrowing choice.

Let μ denote the market's beliefs about the PM's type, i.e. their subjective probability that they are lending to a prudent PM. The model has the following timing.

In period 1:

1. The PM privately observes their type θ.
2. They request a borrowing amount b from the competitive market.
3. Given the borrowing request b and the market's beliefs μ about the PM's type θ, each lender L offers a bond price schedule $q^L(\mu)$.
4. The PM picks among the bond prices on offer, (choosing the highest price on offer,) with $q(\cdot)$ denoting the lending rate offered by the chosen lender.

In period 2:

5. The government defaults on the loan with probability δ^θ, and repays it with the remaining probability.

The equilibrium concept we employ is a Weak Perfect Bayesian Equilibrium, which is defined in our context as follows.

Definition 1 (Weak PBE) *A profile of policymaker strategies $\{\sigma_\theta\}_{\theta\in\{P,E\}}$, a bond price q, and a system of beliefs μ are a* weak perfect Bayesian equilibrium *if they satisfy*

1. *For every θ, every $b \in \sup(\sigma_\theta)$, and every feasible b'*

$$u(y_1 + q(\mu(b))b) + \beta^\theta[(1-\delta^\theta)u(y_2 - b) + \delta^\theta u(y_2)] \geq$$
$$u(y_1 + q(\mu(b'))b') + \beta^\theta[(1-\delta^\theta)u(y_2 - b') + \delta^\theta u(y_2)]$$

2. *For every b, $q(\mu(b)) = \frac{1-\mu(b)\delta^P-(1-\mu(b))\delta^E}{R}$,*
3. *$\mu(b)$ is derived from $\{\sigma_\theta\}_{\theta\in\{P,E\}}$ using Bayes' rule whenever possible.*

The first of these conditions is a set of incentive-compatibility constraints that ensure that neither PM type has an incentive to deviate from their equilibrium strategy, e.g. to mimic the strategy of the other type. The second condition follows from perfect competition in lending markets with risk-neutral lenders or a zero-profit condition. Lenders charge a sovereign spread or risk premium over the risk-free rate of R that exactly compensates for their expected losses due to default. Throughout our analysis, we restrict attention to weak PBE's that satisfy the intuitive criterion, following Cho & Kreps (1987).

We will first analyze a model with full information, in which the markets can observe whether a government is prudent or extravagant. We then analyze a model with asymmetric information.

In each case, we will begin from a model with no fiscal rule and then impose a fiscal rule. To unify notation throughout, we let $b^\theta(\mu; \text{Inf}, \text{Rule})$ denote the optimal debt level for a policymaker with discount factor β^θ, when the lenders assign a probability μ that the policymaker is prudent, when information (*Inf*) is full or asymmetric: Inf $\in \{FI, AI\}$, and the fiscal rule (*Rule*) includes a debt limit or no rule: Rule $\in \{DL, NR\}$. When there is no room for confusion we omit the reference to the information structure and fiscal rule.

For simplicity, we assume that citizens share the same preferences as the prudent policymaker $\beta^C = \beta^P$. We depart from the existing literature on fiscal rules by considering a policymaker that perfectly represents the interests of its citizens. This approach helps highlight the distortions that fiscal rules introduce in this model. The entire positive analysis remains valid regardless of social preferences, and we will explicitly state any normative results that depend on this assumption.

2.1 Full Information - No borrowing limit

In a model with observable types, the financial market knows the default rate and therefore assigns bond price to each PM type that reflects their default probability. A type θ PM can issue bonds at a price q^θ, given by

$$q^\theta = q(\mathbb{1}(\theta = P)) = \frac{1-\delta^\theta}{R}, \tag{5}$$

where $\mathbb{1}(\theta = P)$ is an indicator equaling one if the policymaker is prudent and zero otherwise. The risk premium is unaffected by the loan size, b, because of our simplifying assumption that default probabilities are contingent only on the PM's type, not the amount borrowed. In Section 4, endogenous default makes bond prices decrease in the amount borrowed. The PM of type θ issues b to maximize:

$$U^\theta(b, q^\theta) = u(y_1 + q^\theta b) + \beta^\theta[(1-\delta^\theta)u(y_2 - b) + \delta^\theta u(y_2)].$$

Borrowing $b^\theta(\mathbb{1}(\theta = P); FI, NR)$ satisfies

$$u'(y_1 + q^\theta b^\theta(FI, NR)) = \beta^\theta R u'(y_2 - b^\theta(FI, NR)). \tag{6}$$

The effective interest rate applying to inter-temporal decisions is the same for both types of government and equal to the risk-free rate R because the risk premium exactly offsets default probabilities when types are observable. The bond price still has an endowment effect in period 1, giving the prudent PM more resources in the first period for a given choice of outstanding debt. This gives her a lower incentive to borrow, complementing the the lower desire to borrow due to her greater patience ($\beta^P > \beta^E$). When types are observable, the extravagant PM always borrows more than the prudent one. We summarize this observation as follows:[1]

Observation 1 $b^E(0; FI, NR) > b^P(1; FI, NR)$

The proof of this and all following results are in Appendix C.

This result relies on the assumption that default is exogenous. When the debt burden affects default rates, in Section 2, market discipline through higher sovereign spreads may lead the extravagant government borrow less.

2.2 Full Information - Debt limit

In this section we consider the effect of a debt ceiling.[2] We consider some alternative rules in Sections 4 and 5. For this rule to have any implications, it must be binding for at least one type, so that $\bar{b} \leq b^E(0; FI, NR)$. With a binding constraint, the extravagant type will borrow up to the limit: $b^E(0; FI, DL) = \bar{b}$, while the prudent type will be unaffected unless the debt limit is so tight that it is binding for them as well: $b^P(1; FI, DL) = \min\{b^P(FI, NR), \bar{b}\}$. This leads to:

Observation 2 *When types are observable, the best upper debt limit is given by* $\bar{b} = b^P(0; FI, NR))$.

This observation states that the optimal debt limit will force the extravagant type to borrow exactly as a prudent PM would if it faced the lower bond price q^E, with the higher associated sovereign spread. In other words, the rule aligns the extravagant PM's actions with public preferences, although it takes into account the lower bond price that the extravagant PM faces. This debt limit will not be binding for the prudent PM, who borrows less at the higher bond price it faces ($b^P(1; FI, NR) < b^P(0; FI, NR)$) .

This is the common rationale for a debt limit. Extravagant policymakers are present-biased and a debt limit can align them closer to citizens' long-term interests. The prudent PM is a perfect representative of citizens and there is no reason to alter her behavior. The optimal rule simply forces the extravagant type to behave prudently: to "do the right thing". We have assumed that the prudent government shares citizens' preference. If, instead, the prudent government is present-biased (though less so than the extravagant type), it may be desirable to tighten the rule further to restrict both types.

[1] We drop the market's beliefs to make notation more concise, but it should be clear that the market assigns a probability one to the PM being its actual type. Formally $b^P(FI, NR) = b^P(1; FI, NR)$ and $b^E(FI, NR) = b^E(0; FI, NR)$.

[2] Throughout we assume that the debt limit cannot be circumvented through methods such as creative accounting or shifting debt between national and local governments. Further, we assume that the rule is entirely credible. If the government has ways to circumvent the debt limit, or the market believes that the rule will be loosened in some states of the world, $\bar{b}$ can be viewed as the maximal expected amount of feasible borrowing after all measures to circumvent the rule. See Dovis & Kirpalani (2020) for a theory of fiscal rules with imperfect enforcement.

2.3 Asymmetric information - No debt limit

We now turn to the case where types are privately observed by governments. Lenders must infer default probabilities from policymakers' actions, i.e. their requested borrowing. We restrict attention to equilibria in pure strategies. There are two possible types of pure-strategy equilibria: separating and pooling. We start with the case of separating equilibria. When considering government "types", we find it more useful to think of different potential leaders of a single country than of separate countries. There may be sufficient public information to distinguish between a creditworthy country and a country with a track record of frequent defaults (Reinhart *et al.*, 2003). However, markets may face a greater challenge in evaluating the default propensity of a specific government at a given point in time.

Separating Equilibrium

In a separating equilibrium, the PM's type is revealed to the market and each type is charged a bond price that accords with its objective default probability as in (5). The prudent PM can borrow at a more favorable bond price and she will never want to mimic the extravagant PM, who faces the sovereign spread that could arise. Hence in any separating equilibrium, the extravagant type borrows as it would in the full information equilibrium ($b^E(0; AI, NR) = b^E(0; FI, NR)$).

For equilibrium to be separating, the extravagant policymaker must be satisfied with this strategy and be unwilling or unable to improve his fate by mimicking the prudent type.

$$U^E(b^E(0; AI, NR), q^E) \geq U^E(b^P(1; AI, NR), q^P) \tag{7}$$

The extravagant policymaker faces a tradeoff. He can borrow less than he desires but obtain the higher bond price afforded to prudent policymakers; or he can borrow his desired amount, but at the lower bond price that reflects his higher default probability. This inequality states that the prudent type borrows so little that the extravagant type chooses the latter strategy of borrowing more at the higher sovereign spread.

It is possible that the prudent policymaker's full-information borrowing $b^P(1; FI, NR)$ is already low enough to signal her type, i.e. (7) holds for $b^P(1; AI, NR) = b^P(1; FI, NR)$. Otherwise, she needs to restrain her borrowing to separate from the extravagant type and signal that she is prudent. In a separating equilibrium, this needs to be incentive compatible for the prudent PM, so it must be the case that

$$U^P(b^P(0; FI, NR), q^E) \leq U^P(b^P(1; AI, NR), q^P). \tag{8}$$

That is, the prudent PM will prefer to restrain borrowing to signal that she is prudent over the best outcome she could achieve if she were offered the lower bond price q^E.[3]

To formalize these notions and for future reference, it is useful to introduce additional notation. We denote by $\hat{b}^{\theta}(\mu; b^{\theta'}(\mu'; \text{Inf}, \text{Rule}))$ as the debt level that makes a policymaker of type θ indifferent between being perceived as the prudent type with probability μ and borrowing $\hat{b}^{\theta}$, and being perceived as being the prudent type with probability μ' and borrowing freely at the resultant bond price. As before, the debt level $b^{\theta'}(\mu'; \text{Inf}, \text{Rule})$ is the optimal borrowing of a policymaker of type θ', when the market assigns a probability of μ' that it is prudent, in an equilibrium of type *Inf* with a rule of type *Rule*.

Lemma 1 *In a model of asymmetric information with no fiscal rule, a separating equilibrium exists if and only if $\hat{b}^E(1, b^E(0; FI, NR)) \geq \underline{b}(y_1, q^P)$. Under the intuitive criterion, the unique separating equilibrium (if it exists) is characterized by*[4]

$$b^E(0; AI, NR) = b^E(0; FI, NR)$$

and

$$b^P(1; AI, NR) = \min\{\hat{b}^E(1, b^E(0; FI, NR)), b^P(1; FI, NR)\}$$

In a separating equilibrium, the extravagant type over-borrows due to the standard present-bias problem. However, the prudent PM may *under*-borrow to signal its prudence:

Implication 1 *In a separating equilibrium, the extravagant policymaker over-borrows relative to the social optimum, but the prudent policymaker (weakly) under-borrows.*

This result relies on the alignment between the public's and the prudent PM's preferences. However, this result can occur even if the prudent policymaker is herself present-biased (though more patient than the extravagant one). While such a policymaker would over-borrow with perfect information, she might borrow less than the public would wish to in attempt to signal her relative prudence, under asymmetric information. Thus excessive austerity can arise even if all politicians are present-biased.

Signalling is a common explanation given by governments when implementing austerity programs, e.g. during the global financial crisis. Gaining financial investors' confidence was a central rationale for the UK government's austerity plans following the global financial crisis. The 2011

[3]These are the off-the-equilibrium path beliefs that are most likely to ensure equilibrium existence so that any equilibrium policy must satisfy this condition.

[4]Formally this isn't a unique equilibrium but rather the unique pair of strategies in all separating equilibria. These strategies may be supported with a variety of off-the-equilibrium path beliefs, each formally leading to a different equilibrium.

budget states that "There is a broad international consensus that advanced economies should put in place and begin implementing credible medium-term fiscal consolidation plans this year, in order to underpin market confidence." One can question whether such consensus existed in 2011, but this quote illustrates how financial market confidence has been used as a justification for deficit reduction. Following the fiscal event in the UK in 2022, a majority of UK economists surveyed in the survey of the Centre for Macroeconomics felt that deficit reduction was important to restore the government's credibility (Ilzetzki & Jain, 2022).

Ben Bernanke, although not an advocate of austerity during the financial crisis, argued that "maintaining the confidence of the public and financial markets requires that policy makers begin planning now for the restoration of fiscal balance."[5] Former Treasury Secretary Robert Rubin, writing with later CBO and OMB chair Peter Orszag and Todd Sinai wrote in 2004 that "substantial deficits projected far into the future can cause a fundamental shift in market expectations and a related loss of confidence both at home and abroad" (Rubin *et al.*, 2004). Finance Minister Wolfgang Schäuble defended Euro-area deficit-reduction plans because they would promote "consumer and investor confidence." He states that "governments in and beyond the eurozone need not just to commit to fiscal consolidation... Countries faced with high levels of debt and deficits need to cut expenditures, increase revenues and remove the structural hindrances in their economies, however politically painful... The truth is that governments need the disciplining forces of markets.[6]

The case for confidence-building austerity is equally prevalent in emerging markets. Brazil was on the verge of financial crisis following the election of Lula in 2002. The (first) Lula administration lowered the deficit (of the consolidated public sector) from 4.2% of GDP in 2002 to 2.4% of GDP in 2004 and aimed to bring down the debt-to-GDP ratio by 10 percentage points, largely aimed to calm financial market turmoil.[7] In his address to the IMF in April 2003, Finance Minister Antonio Palocci Filho explained that "To shed concerns regarding debt sustainability, Brazil announced a half percentage point of GDP increase in the primary surplus for 2003, bringing it to 4.25 percent. It reaffirmed its commitment to generate primary surpluses necessary to ensure a steady decline of the debt-to-GDP ratio over the medium term by maintaining a primary surplus target of 4.25 percent of GDP for 2004 and similar indicative target for 2005 and 2006." As a result, "Markets have responded positively to these initiatives. Spreads on Brazilian bonds have been cut from 2,400 to around 900 basis points, and there is still scope for further decline."[8] This impetus to cut deficits to signal fiscal responsibility has implications for the design of fiscal rules as we will shortly see.

[5] Testimony on the Semiannual Monetary Policy Report to the Congress, Before the Committee on Financial Services. U.S. House of Representatives, Washington, DC, July 21, 2009.

[6] Financial Times, September 5, 2011

[7] Source: IMF Article IV 2008.

[8] Statement by Antonio Palocci Filho, Minister of Finance, Brazil, International Monetary and Financial Committee Meeting, Washington, D.C., April 12, 2003.

In our model, markets have incomplete information about governments' impatience and creditworthiness. An existing literature, instead, assumes that *citizens* have incomplete information about the *state of the economy* (Halac & Yared, 2014, 2018; Barnichon & Mesters, 2022). Combining uncertainty on both these dimensions would complicate the analysis substantially. Note that lenders here don't care about the state of the economy per se, but only in so far as it informs them about government's probability of repayment. In our setting, asymmetric information about the current state of the economy y_1 would exacerbate the signalling challenge for governments. On observing a large deficit, markets may now find it difficult to distinguish between a prudent government facing a bad shock to revenues and an exorbitant government facing high revenues.

Absent a lower limit $\underline{g}$ on public goods, Lemma 1 implies that the separating equilibrium characterized here always exists and that it is the unique equilibrium. The prudent type is more patient than the extravagant type and will always be willing to borrow slightly less than the lowest level of debt that the extravagant type is willing to endure to obtain better borrowing terms. Consequently, it is always possible and desirable for the prudent type to signal its prudence. A separating equilibrium fails to materialize only when the prudent type is *unable* to reduce borrowing sufficiently to signal its type because of the minimal public good requirement $\underline{g}$. This explains the condition for equilibrium existence in the lemma. When the austerity required to signal the government's prudence is impossible because of the necessity of minimal public good provisions, pooling equilibria arise.

Pooling Equilibrium

In a pooling equilibrium, the prudent PM is unable to signal her type and the extravagant PM successfully mimics her behavior. The market is unable to update its priors about the PM's type and buys bonds at the price

$$q(\pi) = \frac{1 - \pi\delta^P - (1-\pi)\delta^E}{R}.$$

As we have noted, pooling equilibria exist (if and) only if the prudent PM is constrained by the minimal public good requirement, as summarized in the following lemma.

Lemma 2 *In a model of asymmetric information with no fiscal rule, a pooling equilibrium exists if and only if* $\underline{b}(y_1, q^P) \geq \hat{b}^E(1, b^E(0; FI, NR))$.

As is common in models with asymmetric information, a large set of pooling equilibria may arise in this model, even after eliminating equilibria with the intuitive criterion. In a pooling equilibrium, the market expects to see both the prudent the extravagant PMs borrowing at a specific debt level $b^P(\pi; \text{AI}, \text{NR}) = b^E(\pi; \text{AI}, \text{NR})$. In the zero-probability (off-the-equilibrium-path) event that a government surprises the market with a different request for borrowing, the market concludes that this is an extravagant PM. Fearing the penalizing interest rates they will face if they deviate, both PM types confirm the market's expectations and borrow $b^P(\pi; \text{AI}, \text{NR}) = b^E(\pi; \text{AI}, \text{NR})$. There may be large range of debt levels that can be supported in such an equilibrium.

We characterize the range of pooling equilibria explicitly in Appendix A. Three features of the pooling equilibria are noteworthy. First, the socially optimal debt level may be among the pooling equilibria for some parameter values. The social optimum is the debt level that would be chosen by a social planner that faces the bond price $q(\pi)$ and recognizes that prudent PMs will govern with probability π and extravagant PMs with probability $1 - \pi$. However, there are also parameter values, for which the set of pooling equilibria will all involve over-borrowing relative to the social optimum. For yet other parameter values, all pooling equilibria will involve under-borrowing. It is therefore impossible to make a general statement about whether governments will be borrowing excessively or insufficiently in a pooling equilibrium. And for any given parameter values, it is impossible to evaluate which of the many pooling equilibria will arise.

Second, the set of pooling equilibria may include some very bad outcomes. For example, both PMs may incur as much debt as the extravagant PM would like to borrow or even more. This may seem like an unrealistically perverse outcome, but note that the market cannot infer that a PM is prudent if she borrows less than this extravagant amount. The market has no reason to believe that it isn't facing an extravagant PM if it observes a deviation from the equilibrium strategy, even when applying the intuitive criterion, because *both* PM types would benefit from such a deviation.

There is in fact some empirical evidence for the counterintuitive result that a fiscal limits could increase deficits. Eyraud *et al.* (2018) and Caselli & Wingender (2021) show that fiscal rules in the European Union became "fiscal magnets": Not only did countries with larger deficits decrease deficits towards the 3% target, but also countries with smaller deficits *increased* deficits towards the same target. Simultaneously, EU governments saw a convergence in their borrowing rates. There are some other potential explanations for these phenomena, but the outcomes for EU countries in the period 2000-8 conforms with the predictions of the pooling equilibrium developed here.

Finally, we should note that even in special case where policymakers happen to borrow the "optimal" level of debt, this is an optimum at a the "wrong" borrowing rate. The bond price $q(\pi)$ over-penalizes the prudent PM and under-penalizes the extravagant one relative to their objective default probabilities. In fact, the prudent PM faces a *higher* effective interest rate than does the extravagant PM: They face the same interest rate, but the extravagant PM is more likely to default. The socially optimal debt level takes into account the two different interest rates that might arise and is sub-optimal ex-post regardless of who governs.

2.4 Asymmetric information - Debt limit

We now turn to the implications of a fiscal rule in the form of a debt limit when governments have asymmetrical information about their propensity to borrow and default. We consider a debt limit that is sufficiently tight to bind for the extravagant type, otherwise it has no implications for the equilibrium. We discuss other possible rules in Section 5.

Separating Equilibrium

As before, in a separating equilibrium the two PMs face different bond prices, given in (5). The extravagant PM is constrained by the debt ceiling and borrows to the limit: $b^E(0; AI, DL) = \bar{b} = b^E(0; FI, DL)$. This is precisely what the fiscal rule aims to achieve and counteracts the present-bias problem. However, changing the extravagant PM's behavior also affects the prudent PM's strategy in a separating equilibrium and this may have perverse side-effects. This is the key insight of the paper and is summarized in the following lemma.

Lemma 3 *In a model of asymmetric information with a debt limit, a separating equilibrium exists if and only if* $\underline{b}(y_1, q^P) \leq \hat{b}^E(1, b^E(0; FI, DL))$*. Under the intuitive criterion the unique separating equilibrium (if it exists) is characterized by*

$$b^E(0; AI, DL) = \bar{b} = \hat{b}^E(1, b^E(0; FI, DL))$$

and

$$b^P(1; AI, DL) = \min\{\hat{b}^E(1, b^E(0; AI, DL)), b^P(1; FI, DL)\}$$

Although the debt limit isn't binding for the prudent PM, the rule affects her behavior. The debt limit lowers the extravagant PM's borrowing and makes it more attractive for him to mimic the prudent PM. The prudent PM, attempting to reveal her type, needs to restrain her borrowing even further: It is always the case that $\hat{b}^P(1, b^E(0; AI, DL)) \leq \hat{b}^P(1, b^E(0; AI, NR))$. We summarize this in the following:

Implication 2 *In a separating equilibrium, a debt limit will (weakly) reduce the borrowing of both extravagant and prudent policymakers. This improves citizens' welfare when extravagant policymakers are in power, but hurts citizens' welfare when prudent governments are in power.*

The current literature and policy debate focus on the implications of fiscal rules only when they are binding. We show here that a fiscal rule can distort policy even when it isn't binding. It can induce harsher fiscal austerity policies because the fiscal rule makes it more difficult for prudent governments to show their stripes. The fiscal rule brings the extravagant policymakers' policies closer to those preferred by citizens. But constraining the extravagant PM makes it more attractive for him to mimic the prudent PM to obtain a better bond price. (He has to restrain his borrowing anyways, so he might as well borrow even less and get the lower interest rate while he's at it.)

This is a new informational friction relative to the existing literature. In existing models of fiscal policy with asymmetric information and fiscal rules, governments have private information about the *state of the economy*. Governments always want to over-borrow relative to citizens due to their present bias. The informational problem is that the government may be borrowing a large sum because this is required by economic circumstances, but alternatively it may be incurring large debts because it is present biased. The literature studies how flexible fiscal rules should be in light of this friction. Flexibility allows the government to respond to the state of the economy, on one hand, but it gives the government more leeway to over-borrow, on the other.

In contrast, in the model presented here, policymakers have private information about their propensity to borrow. There is no uncertainty nor private information about the state of the economy. The tradeoff a fiscal rule poses is different here. A fiscal rule limits present-biased policymakers' over-borrowing, but it may also reduce the borrowing of less present-biased policymakers, who are already under-borrowing. For simplicity, we assumed that the prudent policymaker is a perfect agent of citizens' policy preferences. We have noted, however, that under-borrowing could even arise if the prudent policymaker were present-biased relative to citizens. In this case, the prudent policymaker would indeed over-borrow if information were complete, but she could nevertheless under-borrow relative to citizens' preferences under asymmetric information, in the process of signalling her type. The fiscal rule would cause the relatively prudent PM to impose even deeper budget cuts.

Pooling

For completeness, we give conditions for the existence of a pooling equilibrium with a debt limit. As before, pooling equilibria arise (with the intuitive criterion) only when a separating equilibrium isn't feasible. When pooling equilibria exist, there are typically many such equilibria.

Lemma 4 *In a model of asymmetric information with a debt limit, a pooling equilibrium exists if and only if* $\underline{b}(y_1, q^P) \geq \hat{b}^E(1, b^E(0; FI, DL))$.

The main thing to note here is that the debt limit decreases the parameter values in which a separating equilibrium exists and expanded the region where a pooling equilibrium exists. The intuition is that the debt limit requires the prudent PM to reduce her borrowing further to signal her prudence (the value of $\hat{b}^E(1, b^E(0; FI, DL))$ in Lemma 4 is lower than the value of $\hat{b}^E(1, b^E(0; FI, NR))$ in Lemma 2). This makes it more likely that the public good floor $\underline{g}$ is breached and signaling becomes infeasible. The fiscal rule can "jam the signal" and force the prudent PM into a pooling equilibrium.

3 Debt limits: A Cost-Benefit Analysis

The benefit of a debt limit in this model is clear and consistent with conventional wisdom and the existing literature. The debt limit restrains the extravagant policymaker's borrowing (in a separating equilibrium). The costs are twofold. First, the debt limit reduces the prudent PM's borrowing, which is already below the socially optimal debt level. Second, the debt limit makes a pooling equilibrium more likely. As previously noted, there is a large set of possible pooling equilibria, some which are Pareto-inferior to the the separating equilibrium. In all such equilibria, the prudent PM is penalized with a lower bond price than merited by her default probability.

We stack up these costs against the benefits of the debt limit. We begin by analyzing the optimal debt limit in a separating equilibrium.

Optimal Debt Limit

The optimal debt limit from the perspective of citizens trades off the the benefits of reducing debt when the extravagant PM is in power with its costs when the prudent PM is in power.

Observation 3 *The socially optimal level of debt in a separating equilibrium is* $\bar{b} \in (b^P(0; FI, NR), b^E(0; FI, NR))$

This observation states that the optimal debt limit is strictly below the debt level that the extravagant policymaker would choose with full information or in a separating equilibrium without a debt limit. This means that the optimal debt limit does constrain the extravagant PM's borrowing.

However, the observation also states that the optimal debt limit is strictly lower than the debt level that the prudent policymaker would choose if she faced the extravagant PM's bond price q^E. This would be the socially optimal thing to do if constraining the extravagant PM had no impact on the prudent PM's choice. Indeed, this was the optimal debt limit with full information, as noted in Observation 2. Despite this, a social planner should not choose a fiscal rule that imposes this debt level. A naïve debt limit that tries to get policymakers to do the "right thing" isn't good policy.

The proof of this result is in Appendix C, but it is worth explaining why these results hold, as they appear to be rather general. The intuition for these results relies on the envelope theorem. Imagine a debt limit that is just binding for the extravagant PM (it reduces his borrowing only marginally). The extravagant PM is over-borrowing from the citizens' perspective, so that tightening the debt limit increases their utility when he is in power. Hence there is a positive benefit to imposing this debt limit. In a separating equilibrium, the extravagant type is choosing debt optimally from his own perspective. This means that minor deviations imposed by the debt limit have no effect, on the margin, on the extravagant PM's utility. This, in turn, means that the debt limit makes the extravagant type no more tempted to mimic the prudent PM's policy and the prudent PM need not reduce her borrowing any further to signal her type. The implication is that this debt limit imposes no negative externality on the prudent PM or on the public when the prudent PM is in power.

On the other hand, imagine a debt limit that forces extravagant policymakers to choose the naïve constrained social optimum, i.e. the prudent PM's (and public's) preferred debt under full information, but facing the extravagant PM's bond price. This rule would impose the naïve constrained social optimum on the extravagant PM, but isn't binding for the prudent PM in a separating equilibrium. The prudent PM will over-save so as to signal her type and obtain the higher bond price. Now consider a marginal loosening of this debt limit. Minor deviations from the social optimum have negligible social costs so that the cost to the public of allowing the extravagant PM to borrow slightly more when he is in power is approximately zero. On the other hand, the extravagant PM is borrowing more than he would like to due to the debt limit, so that loosening the debt limit increases his utility. This makes the extravagant type materially less inclined to mimic the prudent PM and allows the latter to borrow more while still obtaining the higher bond price. Given that the prudent PM was under-borrowing to begin with, this change improves citizens' welfare when she is in power. Thus a borrowing limit that attempts to impose the seemingly optimal policy on extravagant policymakers is too tight.

Signal Jamming

There is another potential cost to fiscal rules. We have seen that a fiscal rule decreases the parameter space where a separating equilibrium exits. So, for example, there is a smaller range of GDP growth y_2/y_1, for which the prudent PM can signal.

We have already pointed out that this could lead to a wide range of outcomes, some of which are extremely harmful. Therefore, it is impossible to make a general comparison between social welfare in the separating equilibrium and in the wide range of pooling equilibria. There are some pooling equilibria that will dominate the (unique) separating equilibrium and others that are more harmful. However, to gain some intuition about the comparison between the two, it is useful to consider a specific pooling equilibrium. Namely, consider the marginal pooling equilibrium that adds to the set of potential pooling equilibria due to a marginal tightening of the fiscal rule.

In the marginal pooling equilibrium, both PMs obtain a bond price of $q(\pi)$ and borrow $b^P = b^E = \hat{b}^E(\pi, b^E(0; AI, DL))$, i.e. the debt level that leaves the extravagant PM indifferent between borrowing at the debt limit at the more attractive bond price q^P and borrowing $\hat{b}^E$ at the higher bond price $q(\pi)$. There are social costs and benefits when moving from a separating to a pooling equilibrium and these differ depending on the government who is in power. When the extravagant PM is governing, this pooling equilibrium lowers his borrowing. This may be in the the public benefit, because he is over-borrowing in a separating equilibrium. Further, the extravagant PM obtains a lower interest rate. It is easy to show that the public is always better off in this pooling equilibrium than in a separating equilibrium if the extravagant PM is in power: If the extravagant PM is willing to lower his borrowing to obtain improved lending terms, the public should prefer this *a fortiori*, as citizens are more patient than the extravagant PM.

On the other hand, when the prudent PM is in power, the pooling equilibrium reduces public welfare. The prudent PM faces a higher interest rate and borrows more in a pooling equilibrium than in the separating one. It can then be shown that the public is always better off in a separating than in a pooling equilibrium, when the prudent PM is in power.

It is ambiguous whether a separating equilibrium is preferable overall. But the pooling equilibrium always penalizes prudent PMs and rewards extravagant PMs. Perversely, the prudent PM faces a higher effective interest rate than the extravagant one does in a pooling equilibrium. Both PMs face the same bond price, but the extravagant type defaults more frequently. Fiscal rules increase the likelihood of pooling equilibria, which reward impatient and defaulting governments at the expense of more patient and those less prone to default. This could pose perverse incentives that go beyond the analysis of this paper, for example if the degree of fiscal prudence is a matter of public choice or once one takes the economic and social costs of default (Reinhart & Rogoff, 2009; Farah-Yacoub *et al.*, 2022).

4 Endogenous Default

So far, we have assumed that default probabilities are exogenous. This sharpens the model's insights but certainly involves a strong assumption. Avoiding government default is an important rationale for fiscal rules and we now explore how the messages of the previous sections are affected if governments default depends on public debt levels.

To facilitate this analysis, we introduce several additional ingredients to the model. First, the government decides whether it repays or defaults on its debt in period 2. Second, we introduce uncertainty about y_2, revenues in the second period. This creates an ex-ante default probability between zero and one: important to allow an endogenous sovereign spread. Third, we extend the model to a third period. Previously, we imposed that more impatient governments also default more often. Instead, the third period links governments' present bias with their choice to default, allowing default probabilities to arise endogenously in equilibrium. Finally, we assume that default is costly, following a common assumption in the literature and a large body of empirical evidence (Aguiar & Gopinath, 2006; Arellano, 2008).

In the extended framework, the economy now lasts for three periods, $t = 1, 2, 3$. Citizens and the policymaker have preferences over public goods according to the utility function

$$u(g_1) + \beta^\theta u(g_2) + (\beta^\theta)^2 u(g_3),$$

which extends (1) to three periods. The government receives a stream of revenues y_t in each period. Revenues in period 1, y_1, are exogenous. In contrast, revenues in periods 2 and 3 depend on whether the government repays its debt. In each of these periods, y_t denotes the government's income if it repays and $y_t^D = (1-\gamma)y_t$ its income if it defaults, with $0 < \gamma < 1$ representing the economic penalty the government incurs if it defaults on its debt.[9]

Income under full repayment in period 3, y_3, is known in period 1. However, y_2 is revealed to markets and the government only in period 2 and is drawn from a cumulative distribution function $y_2 \sim F(\underline{y}, \bar{y})$, where F has no atoms.

The government chooses its borrowing b in period 1 and it either repays debt in full in period 2 or defaults on the entire sum, so that the budget constraint in period 1 is given by

$$g_1 \leq y_1 + q(b)b,$$

and the period 2 budget constraint is:

$$g_2 \leq \begin{cases} y_2 - b & \text{if the government repays,} \\ (1-\gamma)y_2 & \text{if the government defaults.} \end{cases}$$

Notice that the bond price $q(b)$ is now a function of borrowing b, because higher debt increases the probability of default, which in in turn lowers the bond price offered to the government.

There is no debt in period 3, where the government's budget satisfies

$$g_3 \leq \begin{cases} y_3 & \text{if the government repays,} \\ (1-\gamma)y_3 & \text{if the government defaults.} \end{cases}$$

The timing in this model is the same as in the original model, except that the PM decides in

[9] Aguiar & Amador (2021) discuss how this functional form leads to an unrealistically sharp increase in default probability as debt increases. We nevertheless maintain this assumption to simplify exposition, as we don't conduct a quantitative assessment of our model.

period 2 whether or not to default and then moves on to period 3. Therefore, step 5 of the model becomes:
Period 2:

5. Nature chooses y_2.

6. The government observes y_2 and chooses whether to default on its debt or to repay it in full. It incurs a default penalty of $_2$ if it defaults.

And the model's timing has a final step; in period 3:

7. The government obtains income of y_3 if it repaid in period 2, or $(1-\gamma)y_3$ if it defaulted.

The equilibrium concept is still a Weak Perfect Bayesian Equilibrium and it is solved via backward induction. The policymaker is passive in period 3, merely obtaining payoffs based on its default decision in period 2. In period 2, the type θ PM observes y_2 and, given its choice of b^θ in period 1, it defaults in period 2 if and only if

$$u(y_2 - b^\theta) + \beta^\theta u(y_3) < u((1-\gamma)y_2) + \beta^\theta u((1-\gamma)y_3). \tag{9}$$

That is, the government defaults if the benefit of non-repayment exceeds the default penalty.

Turning to the first period, the PM faces a debt price schedule $q^\theta(b^\theta)$, which depends on market's belief of the PM's type, which maps to its default probability. The PM chooses borrowing b^θ to maximize

$$\begin{aligned} U^\theta(b^\theta, q^\theta(b^\theta)) = u(y_1 + q^\theta(b^\theta)b^\theta) + \beta^\theta \left(\int_{y \notin D^\theta(b^\theta)} u(y - b^\theta) dF(y) + \beta^\theta u(y_3) \right) \\ + \beta^\theta \left(\int_{y \in D^\theta(b^\theta)} u((1-\gamma)y) dF(y) + \beta^\theta u(\gamma y_3) \right), \end{aligned}$$

where $D^\theta(b)$ denotes the values of y_2 for which the PM of type θ defaults in period 2, when it owes b^θ. A Perfect Bayesian Equilibrium in the model with endogenous default can then be defined as follows:

Definition 2 (PBE with Default) *A profile of policymaker strategies $\{\sigma_\theta\}_{\theta \in \{P,E\}}$, where $\sigma_\theta = \{\sigma_\theta^b, \{\sigma_\theta^{D(b)}\}_{b \in R_+}\}$, a bond price schedule $q(b)$, and a system of beliefs μ are a* weak perfect Bayesian equilibrium *if they satisfy*

1. *For every θ, every $b \in \sigma_\theta^b$, every $D(b) \in \sigma_\theta^{D(b)}$, and every feasible b'*

$$u(y_1 + q(b,\mu(b))b) + \beta^\theta \left(\int_{y \notin \sigma_\theta^{D(b)}} u(y-b)dF(y) + \beta^\theta u(y_3) \right)$$
$$+ \beta^\theta \left(\int_{y \in \sigma_\theta^{D(b)}} u((1-\gamma)y)dF(y) + \beta^\theta u((1-\gamma)y_3) \right) \geq$$
$$u(y_1 + q(b',\mu(b'))b') + \beta^\theta \left(\int_{y \notin \sigma_\theta^{D(b')}} u(y-b')dF(y) + \beta^\theta u(y_3) \right)$$
$$+ \beta^\theta \int_{y \in \sigma_\theta^{D(b')}} \left(u(\gamma y) + \beta^\theta u((1-\gamma)y_3) \right) dF(y)$$

2. *For every θ, every $b \in R_+$, and every $y \in \sigma^{D(b)}$*

$$u((1-\gamma)y) + \beta^\theta u((1-\gamma)y_3) \geq u(y-b) + \beta^\theta u(y_3)$$

3. *For every b,*

$$q(b,\mu(b)) = \frac{1 - \mu(b)F(\sigma_P^{D(b)}) - (1-\mu(b))F(\sigma_E^{D(b)})}{R}$$

4. *$\mu(b)$ is derived from $\{\sigma_\theta^b\}_{\theta \in \{P,E\}}$ using Bayes' rule whenever possible.*

This definition is similar to Definition 1, with the added requirement (2) that default be chosen optimally.[10]

We now characterize a few general features of this equilibrium, before turning to the two cases of full and asymmetric information. First, it should be clear that governments default when debt levels are elevated, because the benefit of default is greater then.

Lemma 5 *For all $b < b'$, if default is optimal for b' in some realisations of y_2 for a type θ PM, then default will be optimal for b for the same realisations of y_2 for the type θ PM.*

Second, it is also the case that governments default when revenues are low.

Lemma 6 *For all $y_2 \leq y_2'$, if $y_2' \in \sigma_\theta^{D(b)}$, then $y_2 \in \sigma_\theta^{D(b)}$ for $\theta \in \{P, E\}$.*

This results directly from the fact that the cost of default in the second period γy_2 is increasing in revenues y_2. It follows that for each debt level b we can define a cutoff income level $y(b, \beta^\theta)$, below which a type θ policymaker defaults:

[10] We assume that governments repay when when they are indifferent.

Definition 3 *Denote $y^\theta(b) = y(b, \beta^\theta)$ as the realisation of y_2 for which a type θ PM (one with a discount factor of β^θ) is indifferent between defaulting and repaying. That is $y(b, \beta)$ solves*

$$u((1-\gamma)y(b,\beta^\theta)) - u(y(b,\beta^\theta) - b) = \beta^\theta \left(u(y_3) - u((1-\gamma)y_3)\right)$$

Finally, it is possible to show that more a more present-biased policymaker will default with higher probability, and therefore the extravagant type defaults more often than the prudent type.

Lemma 7 *For any debt level b, $y^P(b) < y^E(b)$. That is, if a prudent PM defaults, an extravagant PM will do so as well.*

Lemmas (5) to (7) repeat the standard results from the literature that sovereign default is more likely when debt is high and income is low. In addition, the frequency of default is decreasing in governments' patience because patient governments put a higher weight on future penalties to default.

4.1 Full information

We begin with the case of full information. When the PM's type is observable by markets, the PM's default probability is known and markets assign a bond price to each PM reflecting this probability. A type θ PM will default in period 2 if $y_2 \leq y^\theta(b)$. This occurs with an ex-ante probability $F(y^\theta(b))$. Therefore, a PM of type θ issues bonds b at a price $q^\theta(b)$, given by

$$q^\theta(b) = q(b, \mathbb{1}_{\theta=P}) = \frac{1 - F(y^\theta(b))}{R}.$$

Moving to period 1, a type θ PM issues debt b to maximize:

$$\begin{aligned} U^\theta(b, q^\theta(b)) = u(y_1 + q^\theta(b)b) + \beta^\theta \left(\int_{y^\theta(b)}^{\bar{y}} u(y - b^\theta) dF(y) + \beta^\theta u(y) \right) \\ + \beta^\theta \left(\int_{\underline{y}}^{y^\theta(b)} u((1-\gamma)y) dF(y) + \beta^\theta u((1-\gamma)y_3) \right), \end{aligned}$$

Borrowing $b^\theta = b^\theta(\mathbb{1}_{\theta=P}; FI, NR)$ satisfies

$$u'(y_1 + q^\theta(b^\theta)b^\theta) \left(q^\theta(b^\theta) + \frac{\partial q^\theta(b^\theta)}{\partial b^\theta} b^\theta \right) = \beta^\theta \int_{y^\theta(b^\theta)}^{\bar{y}} u'(y - b^\theta) dF(y)$$

There are two forces that determine borrowing in this model. As in the baseline model, there is a standard intertemporal motivation to shift public good spending across periods in accordance with the policymaker's time preferences. The integral on the right-hand side reflects the probability of repayment, mirroring the probability δ in the baseline model. In addition, when default is endogenous, the bond price is also endogenous, and the PM takes into account that issuing more debt increases borrowing costs (reduces the bond price). This increases the cost of borrowing on the margin and therefore dampens borrowing motivations for all PM types. [11]

When we assumed exogenous default probabilities, Observation 1 stated that the extravagant PM always borrows more than the prudent one. This is no longer true when default probabilities are determined in equilibrium. Here, the extravagant PM not only obtains a lower bond price at a given debt level ($q^E(b) < q^P(b)$). But also, the extravagant PM's sovereign spread may be more sensitive to borrowing than the prudent PM's: $\frac{\partial q^E(b)}{\partial b} < \frac{\partial q^P(b)}{\partial b}$. Therefore, there are circumstances when (parameter values such that) the prudent PM borrows *more* than the extravagant type. This is because there are now two competing forces: On one hand, the extravagant type is more present-biased and more inclined to borrow at a given bond price; on the other hand, it pays a greater penalty–a higher sovereign spread.

In any full-information equilibrium where the extravagant PM borrows more, the insights of Section 2 hold exactly. A fiscal rule that is binding for the extravagant type but not the prudent type is always welfare improving. Such a rule now has two advantages. First, it lowers the extravagant type's borrowing, aligning it more closely with citizens' preferences. Second, this lower borrowing now also lowers borrowing costs (increases the bond price). As in the baseline model, the optimal fiscal rule sets $b^E = b^P(0; FI, NR)$, i.e. forces the extravagant government to borrow as though it were prudent.

In contrast, a fiscal rule may be counterproductive when the extravagant PM borrows less $b^E(1; FI, NR) < b^P(0; FI, NR)$. Now, a simple limit on debt, borrowing, or debt-to-GDP will restrict the prudent PM more than the extravagant one. Market discipline is already inducing the extravagant PM to borrow less. Conversely, the market is rewarding the prudent PM for her perceived fiscal responsibility and a fiscal rule denies her the rewards to her prudence.

Hatchondo *et al.* (2022a,b) have proposed an alternative to the simple debt limit we explore here: one that puts a cap on sovereign spreads rather than on debt. Such a rule requires governments to reduce borrowing until a specific bond price, rather than debt limit, is achieved. Such a rule would be effective here too and could replicate the results of Section 2. Even when the extravagant type borrows less, it has a lower equilibrium bond price (which is precisely what causes it to borrow less). The mapping from debt to bond prices in $q^\theta(b)$ allows a benevolent planner to set a bond price floor that re-aligns the extravagant PM's borrowing with public preferences. The rule

[11] There is potentially a third factor determining optimal debt. An increase in debt increases in the probability that the PM defaults in period 2, i.e. decreases the income level of default $y^{theta}(b)$. However, default in period 2 is chosen optimally and the PM is indifferent between defaulting and repaying at that marginal income level. Therefore increasing the probability of default doesn't affect the PM's payoff on the margin.

would not be able to avoid the penalty the extravagant government receives for its higher probability of default. However, the the optimal rule would account for the fact that the extravagant type is more likely to default and would restrict its borrowing to contain this risk. In fact, it can be shown that an optimal restriction on sovereign spreads would lead the extravagant government to borrow less than the prudent type, so that market discipline and and a fiscal rule involving a cap on sovereign spreads complement each other.

4.2 Asymmetric Information

When markets cannot observe the government's type, they do not know its default probability. Here, as in the baseline model, the prudent type may reduce its borrowing to signal its prudence. Two potential equilibria emerge: a separating equilibrium, where the prudent PM successfully conveys this information; and a pooling equilibrium where the extravagant policymaker successfully mimics the prudent type, to obtain a lower sovereign spread.

The equilibria are analyzed in Appendix B. The analysis demonstrates that all the results from Section 2 remain valid when default is an endogenous choice. Concretely, without a lower bound on public goods $\underline{g}$, there is a unique, separating, equilibrium. In this equilibrium, the extravagant policymaker behaves as he would under full information, while the prudent PM borrows less to signal her type. Strikingly, asymmetric information leads the prudent PM to borrow less than the extravagant one, even if it would have borrowed more with full information. While the extravagant PM over-borrows due to present bias, the informational friction causes the prudent type to under-borrow to signal to the market.

Similar to the benchmark model, a fiscal rule poses a tradeoff. On one hand, it restrains the extravagant PM; on the other hand, it complicates signalling and drives the prudent PM to impose even harsher austerity than would be necessary absent the debt limit. This poses a problem even if the fiscal rule reduces the risk premia of both types of government. The extravagant type tends to over-borrow and defaults too frequently than citizens would prefer, so lowering its borrowing and sovereign spread through a debt limit benefits the public. In contrast, the prudent PM's policy preferences are aligned with those of citizens. She defaults only when it serves the public interest. With full information, she then borrows the socially desirable amount and assumes appropriate level of risk. Under asymmetric information, she under-borrows and takes on insufficient risk, in order to signal her prudence. Imposing a debt limit on top of this market discipline may lower the sovereign spread further, but this ultimately harms the public interest, which calls for a higher level of risk. While this logic assumes that the prudent PM is no less patient than the public, it may still hold even if the prudent PM is present biased (although less so than the extravagant PM).

With full information, we noted that the fiscal rule could be improved by considering sovereign spreads. However, such a rule cannot resolve the problem of excessive austerity arising due to asymmetric information. There is a one-to-one mapping between the extravagant PM's borrowing, its default risk, and the sovereign spread it faces. Setting a debt limit that targets a sovereign spread rather than a borrowing level encourages the extravagant PM to mimic the prudent one to the same extent. In fact, a cap on spreads has the exact same implication in this setting as a cap on debt. There are other merits to a spread-based fiscal rule, as discussed in Hatchondo *et al.* (2022a,b), but such a rule doesn't resolve the information transmission problem introduced here.

5 Policy Implications

At the heart of this paper is the tradeoff surrounding fiscal rules that limit government borrowing. While such rules can restrain the excessive borrowing tendencies of present-biased governments, they can also increase the risk that governments impose excessive austerity to mollify market concerns. This dilemma differs from the typical critique leveled at fiscal rules–that they lack the necessary flexibility to increase borrowing during crises.

We noted before that evaluating the interaction of these two critiques requires greater modeling complexity. With this caveat in mind, proposals for more flexible fiscal rules do not appear to tackle head-on the informational friction we investigate here. Certainly, there may be circumstances where a flexible fiscal rule gives an escape valve of extra borrowing for a slowing economy. Insofar as extravagant policymakers are more likely to use these escape clauses, these may improve information transmission relative to a hard fiscal rule. Similarly, the greater flexibility of fiscal norms (Blanchard *et al.*, 2021) may give greater leeway for prudent governments to distinguish themselves from extravagant ones. But these solutions are tailored to address a different concern and address the problems investigated here only indirectly.

In addressing the informational friction studied here, it is important to note that markets are only concerned about a government's propensity to default, not it present-bias per se. Therefore, it is not necessary to reveal the government's "type", only its default probability. Rules that anchor market expectations about sovereign default, and detach them from the nature of the specific policymaker, could prove more effective. To see this in the context of the model, consider a rule that specifies exactly in which states of the world (values of y_2) the government is authorized to default. For this illustration, it is important that the rule be transparent, not necessarily optimally designed. With such a rule, the government's present bias is immaterial for its ex-ante probability of default and all government types obtain the same bond price schedule. Extravagant policymakers then have no incentive to mimic prudent ones because doing so no longer moves them to a better schedule. Consequently, prudent policymakers no longer need to impose excessive austerity to signal their prudence. Admittedly, governments may still have the incentive to signal their prudence to *voters*, but no longer to markets.

Importantly, with a rule governing default in place, a debt limit or a cap on spreads is no longer harmful. The debt limit can now restrain extravagant policymakers, aligning them more closely with citizens' long-term interests. But having addressed the informational friction with the default rule, the debt rule no longer induces prudent governments to resort to excessive austerity to prove their prudence.

How would such a default rule look in practice? Several countries have enshrined deficit limits in their constitutions (Eyraud *et al.*, 2018) or have committed to deficit rules in international agreements (e.g. EU member states). Our model suggests that these should be complemented with rules governing public debt repayment. Such rules could require the government to repay its debts unless specific measurable forms of duress trigger an escape clause. Constitutional requirements for debt repayment aren't unheard off. Some legal scholars (citing Perry vs. the United States, 1935) claim that repayment of public debt is enshrined in Section 4 of the 14$^{\text{th}}$ amendment to the US constitution, which states

> The validity of the public debt of the United States, authorized by law, including debts incurred for payment of pensions and bounties for services in suppressing insurrection or rebellion, shall not be questioned.

It may be impractical for countries with a history of default to go as far as the US constitution, but legislation that increases transparency as to what would trigger default would be a step in this direction. EU member states would particularly benefit from clarifying the status of their public debts. EU treaties do not preclude member states' default. And while the EU is forbidden from bailing members out, this interdiction may have been breached during the Eurozone crisis of the previous decade, and the ECB may have added to this ambiguity though its Outright Monetary Transactions.

Collective Action Clauses (CACs) may offer a partial solution for countries unable or unwilling to legislate precise conditions for sovereign default. While CACs do not stipulate the conditions for it issuer to default, they can anchor expectations of the circumstances that lead to debt restructuring. If these conditions are seen as less linked to the prudence of the policymaker in power, they may reduce the asymmetric information about default probabilities and mitigate the informational frictions discussed here.

6 Concluding Remarks

This paper studies the interaction between fiscal discipline achieved through market mechanisms and discipline imposed by external rules. Market forces impose fiscal responsibility on governments. We have shown that the resulting rectitude could be so strict that it may cause a benevolent government to over-save (adopt austerity policies) to signal its fiscal prudence. Ignoring the discipline imposed by markets and policymakers' signalling incentives, it is correct that a fiscal rule should merely attempt to replicate socially desirable policies. However, in the presence of market discipline, imposing naïve fiscal rules can lead to unintended and undesirable consequences. These outcomes may occur even when the fiscal rule isn't binding and seemingly has no effect. Fiscal rules may make it more difficult for prudent policymakers to signal their responsibility and may incentivize them to impose even harsher austerity policies. Alternatively, they may conclude that signaling their fiscal responsibility is too costly and accept the higher market interest rates that result.

Our analysis suggests that most forms of debt and deficit limits complicate government's attempts to prove their rectitude to markets. Instead, we have suggested that clearer rules governing default can mitigate asymmetric information about default probabilities and belie governments' need to signal their prudence.

A few caveats are due. First, we have assumed that the rules are fully enforceable. In many cases, creative accounting and shifting expenditures across levels of the federal government make fiscal rules no more than a line in the sand. It would be interesting to investigate how incomplete information about the government's budget affects the signalling problems discussed here.

Similarly, the analysis assumes that the fiscal rule is external to the policy-making process. This is perhaps relevant for a European government in face of the external EU fiscal rules. This may also be relevant in cases where the fiscal rule requires a super-majority to reverse, for example if it is constitutionally enshrined. But in most cases, rules are set by the very policymakers that face them. For example, UK fiscal rules require a simple majority to change and this is the same majority required to pass any individual budget. It is an interesting question, worthy of future research, whether and how fiscal rules have any bite in these circumstances. One dimension that is related to the analysis we conducted here is the possibility that introducing, enforcing, violating, and overturning fiscal rules are themselves symbolic acts that have some signalling value.

References

AGUIAR, MARK, & AMADOR, MANUEL. 2021. *The Economics of Sovereign Debt and Default*. Princeton: Princeton University Press.

AGUIAR, MARK, & GOPINATH, GITA. 2006. Defaultable debt, interest rates and the current account. *Journal of International Economics*, **69**(1), 64–83. Emerging Markets.

ALESINA, ALBERTO, & TABELLINI, GUIDO. 1990. A positive theory of fiscal deficits and government debt. *The Review of Economic Studies*, **57**(3), 403–414.

AMADOR, MANUEL, & PHELAN, CHRISTOPHER. 2021. Reputation and Sovereign Default. *Econometrica*, **89**(4), 1979–2010.

ARELLANO, CRISTINA. 2008. Default Risk and Income Fluctuations in Emerging Economies. *American Economic Review*, **98**(3), 690–712.

ARNOLD, NATHANIEL G, BALAKRISHNAN, RAVI, BARKBU, BERGLJOT B, DAVOODI, HAMID R, LAGERBORG, ANDRESA, LAM, WAIKEI R, MEDAS, PAULO A, OTTEN, JULIA, RABIER, LOUISE, ROEHLER, CHRISTIANE, *et al.* 2022. Reforming the EU Fiscal Framework: Strengthening the Fiscal Rules and Institutions. *IMF Departmental Papers*, **2022**(014).

AZZIMONTI, MARINA, BATTAGLINI, MARCO, & COATE, STEPHEN. 2016. The costs and benefits of balanced budget rules: Lessons from a political economy model of fiscal policy. *Journal of Public Economics*, **136**, 45–61.

BARNICHON, RÉGIS, & MESTERS, GEERT. 2022. Reconciling Fiscal Ceilings with Macro Stabilization.

BESLEY, TIMOTHY. 2007. *Principled Agents?: The Political Economy of Good Government*. OUP Catalogue, no. 9780199283910. Oxford University Press.

BESLEY, TIMOTHY, & SMART, MICHAEL. 2007. Fiscal restraints and voter welfare. *Journal of Public Economics*, **91**(3-4), 755–773.

BILBIIE, FLORIN, MONACELLI, TOMMASO, & PEROTTI, ROBERTO. 2021. Fiscal policy in Europe: controversies over rules, mutual insurance, and centralization. *Journal of Economic Perspectives*, **35**(2), 77–100.

BLANCHARD, OLIVIER, LEANDRO, ALVARO, & ZETTELMEYER, JEROMIN. 2021. Redesigning EU fiscal rules: From rules to standards. *Economic Policy*, **36**(106), 195–236.

BULOW, JEREMY, & ROGOFF, KENNETH. 1988. Multilateral negotiations for rescheduling developing country debt: a bargaining-theoretic framework. *IMF Staff Papers*, **35**(4), 644–657.

BULOW, JEREMY, & ROGOFF, KENNETH. 1989a. A constant recontracting model of sovereign debt. *Journal of Political Economy*, **97**(1), 155–178.

BULOW, JEREMY, & ROGOFF, KENNETH. 1989b. Sovereign Debt: Is to Forgive to Forget? *American Economic Review*, **79**(1), 43–50.

BULOW, JEREMY, & ROGOFF, KENNETH. 1990. Cleaning up third world debt without getting taken to the cleaners. *Journal of Economic Perspectives*, **4**(1), 31–42.

BULOW, JEREMY, & ROGOFF, KENNETH. 1991. Sovereign debt repurchases: No cure for overhang. *The Quarterly Journal of Economics*, **106**(4), 1219–1235.

BULOW, JEREMY, ROGOFF, KENNETH, & DORNBUSCH, RUDIGER. 1988. The buyback boondoggle. *Brookings Papers on Economic Activity*, **1988**(2), 675–704.

BULOW, JEREMY, ROGOFF, KENNETH, BEVILAQUA, AFONSO S, COLLINS, SUSAN, & BRUNO, MICHAEL. 1992. Official creditor seniority and burden-sharing in the former Soviet bloc. *Brookings Papers on Economic Activity*, **1992**(1), 195–234.

CAMPANTE, FILIPE, STURZENEGGER, FEDERICO, & VELASCO, ANDRES. 2021. Fiscal policy II: the long-run determinants of fiscal policy.

CASELLI, FRANCESCA, & WINGENDER, PHILIPPE. 2021. Heterogeneous effects of fiscal rules: The Maastricht fiscal criterion and the counterfactual distribution of government deficitsâ°. *European Economic Review*, **136**(C).

CHO, IN-KOO, & KREPS, DAVID M. 1987. Signaling Games and Stable Equilibria. *The Quarterly Journal of Economics*, **102**(2), 179–221.

DOVIS, ALESSANDRO, & KIRPALANI, RISHABH. 2020. Fiscal Rules, Bailouts, and Reputation in Federal Governments. *American Economic Review*, **110**(3), 860–88.

EYRAUD, LUC, DEBRUN, XAVIER, HODGE, ANDREW, LLEDÓ, VICTOR, & PATTILLO, CATHERINE. 2018 (Aug.). *Second-Generation Fiscal Rules: Balancing Simplicity, Flexibility, and Enforceability* . IMF Staff Discussion Note SDN/18/04. The International Monetary Fund.

FARAH-YACOUB, JUAN P., GRAF VON LUCKNER, CLEMENS MATHIS HENRIK, RAMALHO, RITA, & REINHART, CARMEN M. 2022 (Aug.). *The Social Costs of Sovereign Default*. Policy Research Working Paper Series 10157. The World Bank.

GIBERT, ANNA. 2022. Signalling creditworthiness with fiscal austerity. *European Economic Review*, **144**, 104090.

HALAC, MARINA, & YARED, PIERRE. 2014. Fiscal rules and discretion under persistent shocks. *Econometrica*, **82**(5), 1557–1614.

HALAC, MARINA, & YARED, PIERRE. 2018. Fiscal Rules and Discretion in a World Economy. *American Economic Review*, **108**(8), 2305–34.

HATCHONDO, JUAN CARLOS, MARTINEZ, LEONARDO, & ROCH, FRANCISCO. 2022a. Fiscal Rules and the Sovereign Default Premium. *American Economic Journal: Macroeconomics*, **14**(4), 244–73.

HATCHONDO, JUAN CARLOS, MARTINEZ, LEONARDO, & ROCH, FRANCISCO. 2022b. Numerical fiscal rules for economic unions: The role of sovereign spreads. *Economics Letters*, **210**, 110168.

ILZETZKI, ETHAN. 2021. Fiscal rules in the European Monetary Union. *VoxEU*, Jun.

ILZETZKI, ETHAN, & JAIN, SURYAANSH. 2022. Addressing the UK's public finances after the mini-budget crisis. *VoxEU*, Dec.

PIGUILLEM, FACUNDO, & RIBONI, ALESSANDRO. 2021. Fiscal rules as bargaining chips. *The Review of Economic Studies*, **88**(5), 2439–2478.

REINHART, CARMEN M., & ROGOFF, KENNETH S. 2009. *This Time Is Different: Eight Centuries of Financial Folly*. Economics Books, no. 8973. Princeton University Press.

REINHART, CARMEN M., ROGOFF, KENNETH S., & SAVASTANO, MIGUEL A. 2003. Debt Intolerance. *Brookings Papers on Economic Activity*, **34**(1), 1–74.

ROGOFF, KENNETH. 1985. The optimal degree of commitment to an intermediate monetary target. *The Quarterly Journal of Economics*, **100**(4), 1169–1189.

ROGOFF, KENNETH. 1990. Equilibrium Political Budget Cycles. *The American Economic Review*, **80**(1), 21–36.

ROGOFF, KENNETH, & SIBERT, ANNE. 1988. Elections and macroeconomic policy cycles. *The Review of Economic Studies*, **55**(1), 1–16.

RUBIN, ROBERT E, ORSZAG, PETER R, & SINAI, ALLEN. 2004. Sustained budget deficits: longer-run US economic performance and the risk of financial and fiscal disarray. *In: AEA-NAEFA Joint Session, Allied Social Science Associations Annual Meetings, The Andrew Brimmer Policy Forum,ââNational Economic and Financial Policies for Growth and Stability.ââJanuary*, vol. 5. Citeseer.

THYGESEN, NIELS, SZCZUREK, MATEUSZ, BORDIGNON, MASSIMO, DEBRUN, XAVIER, & BEETSMA, ROEL. 2022. Making the EU and national budgetary frameworks work together. *VoxEU*, September.

YARED, PIERRE. 2019. Rising government debt: Causes and solutions for a decades-old trend. *Journal of Economic Perspectives*, **33**(2), 115–40.

Zettelmeyer, Jeromin, Véron, Nicolas, Weder di Mauro, Beatrice, Rey, Hélène, Schnabel, Isabel, Martin, Philippe, Pisani-Ferry, Jean, Fuest, Clemens, Gourinchas, Pierre-Olivier, Fratzscher, Marcel, Farhi, Emmanuel, Enderlein, Henrik, Brunnermeier, Markus, & Bénassy-Quéré, Agnès. 2018. How to reconcile risk sharing and market discipline in the euro area. *VoxEU*, January.

A Characterization of Pooling Equilibria

We now give an exact characterization of which pooling equilibria that survive the intuitive criterion given the minimum level of government spending, $\underline{g}$, and fiscal rule, $\bar{b}$. Note that for $\bar{b}$ sufficiently high this captures the case of in which there is no fiscal rule.

Note that $b^P(0; FI, NR) > b^P(\pi; FI, NR)$, since $q(\pi) > q^E$. If not, then

$$\beta^P Ru'(y_2 - b^P(\pi; FI, NR)) = u'(y_1 + q(\pi) b^P(\pi; FI, NR)) \tag{10}$$

$$< u'(y_1 + q^E b^P(0; FI, NR)) = \beta^P Ru'(y_2 - b^P(0; FI, NR)) \leq \beta^P Ru'(y_2 - b^P(\pi; FI, NR)), \tag{11}$$

where the equalities follows from first order conditions, and the inequalities follows since u is concave, $b^P(0; FI, NR) \leq b^P(\pi; FI, NR)$ and $q(\pi) > q^E$.

As it will clearly not be optimal to impose a debt limit that restricts the debt such much such that it restricts the borrowing of the prudent type, when facing the worst possible bond price. Hence through out in this section we assume that $\bar{b} \geq b^P(0; FI, NR)$.

Let $\underline{g}$ and $\bar{b}$ be given. Denote by $b^P(0) = \max\{b^P(0; FI, NR), \underline{b}(y_1, q^E)\}$, and $b^E(0) = \min\{b^E(0; FI, NR), \bar{b}\}$.

Note, that when the minimum level for government spending restricts the choices of the Prudent government, when she is perceived as the Extravagant type ($b^P(0) > b^P(0; FI, NR)$), then highest level the Prudent government will be willing to choose when faced with the bond price of $q(\pi)$ increases ($\hat{b}^P(\pi, b^P(0)) > \hat{b}^P(\pi, b^P(0; FI, NR))$ for $\hat{b}^P(\pi, b^P(0)) > b^P(0)$, and $\hat{b}^P(\pi, b^P(0; FI, NR)) > b^P(0; FI, NR)$). Similarly, when the borrowing limit restrict the Extravagant government when perceived as the extravagant type, then the lowest level the extravagant government is willing to choose when faced with the bond price of $q(\pi)$ decreases ($b^E(0) < b^E(0; FI, NR)$ then $\hat{b}^E(\pi, b^E(0)) > \hat{b}^E(\pi, b^E(0; FI, NR))$ for $\hat{b}^E(\pi, b^E(0)) < b^E(0)$, and $\hat{b}^E(\pi, b^E(0; FI, NR)) < b^P(0; FI, NR)$).

By the same arguments as in the proof of Lemma 2 the range of possible bond levels that can be sustained in a pooling equilibrium with $\underline{g}$ and $\bar{b}$ is given by

$$A(\underline{g}, \bar{b}) = \left[\hat{b}^E(\pi, b^E(0)), \hat{b}^P(\pi, b^P(0))\right] \tag{12}$$

where $\hat{b}^E(\pi, b^E(0))$ is the lowest downward deviation that the Extravagant government will be willing to choose when faced with the bond price $q(\pi)$ ($\hat{b}^E(\pi, b^E(0)) < b^E(0)$) and $\hat{b}^P(\pi, b^P(0))$ is highest upward deviation that the Prudent government will be willing to choose when faced with the bond price of $q(\pi)$ instead of q^E ($\hat{b}^P(\pi, b^P(0)) > b^P(0)$). Furthermore, let

$$B(\underline{g}, \bar{b}) = [\underline{b}(y_1, q(\pi)), \bar{b}] \tag{13}$$

be the set of bond levels that are feasible when faced with the bond price of $q(\pi)$.

Thus, for $b^*(\pi)$ to be the bond level to be sustained as the bond level requested in a pooling equilibrium, then $b^*(\pi) \in A(\underline{g}, \bar{b}) \cap B(\underline{g}, \bar{b})$.

So far we have been silent about the off-equilibrium beliefs. Suppose that $b^*(\pi) \in A(\underline{g}, \bar{b}) \cap B(\underline{g}, \bar{b})$ is the bond-level requested by both types of governments in equilibrium. Let the beliefs be as follows:

$$\mu(b) = \begin{cases} \pi & \text{if } b = b^*(\pi) \\ 0 & \text{if } b > \hat{b}^P(1, b^*(\pi)) \\ 0 & \text{if } b \neq b^*(pi), b \geq \hat{b}^E(1, b^*(\pi)) \text{ and } b \leq \hat{b}^P(1, b^*(\pi)) \\ 1 & \text{if } b < \hat{b}^E(1, b^*(\pi)) \end{cases} \tag{14}$$

where the $\mu(b^*(\pi)) = \pi$ is given by Bayes rule, $\mu(b) = 0$ is $b > \hat{b}^P(1, b^*(\pi))$ and $\mu(b) = 1$ if $b < \hat{b}^E(1, b^*(\pi))$ follows from the restrictions imposed by the intuitive criterion. For the remaining of equilibrium values of b the intuitive criterion has no bite, so we assume the beliefs that will make deviations least attractive.

If $\hat{b}^E(1, b^*(\pi)) > \underline{b}(y_1, q^P)$, then the Prudent government will deviate to $b = \hat{b}^E(1, b^*(\pi)) - \varepsilon$ for $\varepsilon > 0$ small. Because of the restrictions imposed by the intuitive criterion, and sequential rationally, the bond price for b will be q^P, and thus will be a profitable deviation for the Prudent government. Such a deviation is not possible if $\hat{b}^E(1, b^*(\pi)) \leq \underline{b}(y_1, q^P)$. Since any bond level b for which the intuitive criterion would imply that $\mu(b) = 1$ are not feasible.

We conclude that the set of pooling equilibria are characterized by $b^*(\pi) \in A(\underline{g}, \bar{b}) \cap B(\underline{g}, \bar{b})$ for which $\hat{b}^E(1, b^*(\pi)) \leq \underline{b}(y_1, q^P)$.

To gain a better understanding of the set of pooling equilibria, we consider the effects of changes to $\underline{g}$ and $\bar{b}$.

First consider the comparative statics of a marginal increase in the minimum level of government spending $\underline{g}$:

$$\frac{\partial b^P(0)}{\partial \underline{g}} = \begin{cases} 0 & \text{if } b^P(0) \geq \underline{b}(y_1, q^E) \\ \frac{R}{1-\delta^E} > 0 & \text{if } b^P(0) < \underline{b}(y_1, q^E) \end{cases}, \tag{15}$$

and

$$\frac{\partial \underline{b}(y_1, q(\pi))}{\partial \underline{g}} = \frac{R}{1 - \pi\delta^P - (1-\pi)\delta^E} \in \left(0, \frac{R}{1-\delta^E}\right). \tag{16}$$

When $\frac{\partial b^P(0)}{\partial \underline{g}} > 0$, an increase in the minimum level of required government spending increases $A(\underline{g}, \bar{b})$ as the upper bound increases. This tends to increase the set of possible pooling equilibria on the margin. On the other hand, it decreases $B(\underline{g})$ so that the set of feasible pooling equilibrium may decrease.

Next, we consider how changes to the debt limit affects the set of pooling equilibria that can be sustained in equilibrium. The debt limit has a direct effect as follows:

$$\frac{\partial b^E(0)}{\partial \bar{b}} = \begin{cases} 0 & \text{if } b^E(0) < \bar{b} \\ -1 < 0 & \text{if } b^E(0) < \bar{b} \end{cases}. \tag{17}$$

This in turn decreases the lowest debt level the Extravagant government is willing to choose if she is perceived as the Prudent type for any bond levels when faced with the bond price $q(\pi)$. Thus, it increases the range of pooling equilibria possible.

In general, it is not possible to determine whether $b^P(\pi; FI, NR)$ is lower or higher than $\hat{b}^E(\pi, b^E(0))$. If $\hat{b}^E(\pi, b^E(0)) > b^P(\pi; FI, NR)$, then lowering the debt limit sufficiently will make it possible to sustain $b^P(\pi; FI, NR)$ in a pooling equilibrium. If $\hat{b}^E(\pi, b^E(0)) < b^P(\pi; FI, NR)$, then relaxing the the debt limit, will exclude suboptimal pooling equilibria. Note, however, whether or not the socially optimal level is feasible depends on the minimum level of government spending.

B Characterization of Equilibria with Endogenous Default

B.1 Unobservable types - Separating Equilibrium

Notice that when the PM's type is their private information, the market forms beliefs about the PM's type for every value of b, $\mu(b)$. Let $q(b) = q(b, \mu(b))$ denote the bond price schedule that a policymaker faces if it issues a quantity b of bonds and faces a market belief function $\mu(b)$. The following lemma notes that PMs prefer borrowing less the more patient they are, if facing a bond price schedule $q(b)$.

Lemma 8 *Given a bond price schedule $q(b)$ the marginal costs (benefits) from decreasing the debt level is decreasing (increasing) in β.*

Proof. All proofs are found in Appendix C ■

That is, downwards deviations are cheaper for a prudent PM than for an extravagant PM, when they face the same bond price schedule.

Since $y(b, \beta^P) \leq$y(b,β^E) (Lemma 7), we have $q^P(b) \geq q^E(b)$ for any b. The prudent PM will therefore never want to mimic the extravagant PM, and the bond price schedule faced by the extravagant PM is the worst that could arise. Hence in any separating equilibrium, we have $b^E(0; AI, NR) = b^E(0; FI, NR) = b^E$. A necessary condition for a separating equilibrium is:

$$U^E(b^E(0; AI, NR), q(b^E(0; AI, NR), 0)) \geq U^E(b^P(1; AI, NR), q(b^P(1; AI, NR), 1)), \tag{18}$$

that is, the extravagant PM doesn't want to mimic the prudent one. Notation is slightly complicated with endogenous default, because the set of debt levels that satisfy this incentive compatibility constraint may be disconnected. Let $A(NR)$ give the set of debt levels that are below the full-information debt level of the extravagant PM that satisfy his incentive compatibility constraint 18, if chosen by the prudent PM: $A(NR) = \{b < b^E(0; FI, NR) | U^E(b^E(0; FI, NR), q(b^E(0; FI, NR), 0)) \geq U^E(b, q(b, 1))\}$. In other words, this is the set of debt levels, low enough to signal that the policymaker is prudent. Further, we denote by $\hat{b}^E(NR)$ the highest level of debt within this set, i.e. the highest debt level that is nevertheless low enough to signal that the policymaker is prudent: $\hat{b}^E(NR) = \max_{b \in A(NR)} b$. Finally, let $\underline{b}(y_1, P)$ give the minimal borrowing the prudent PM requires in order to provide the minimal public good requirement $\underline{g}$: $\underline{b}(y_1, P) = \min_b \{b : bq^P(b) \geq \underline{g} - y_1\}$. With this notation in mind, we can characterize the conditions for a separating equilibrium to exist, as follows:

Lemma 9 *In a model of asymmetric information with no fiscal rule and endogenous default rate, a separating equilibrium exists if and only if $\hat{b}^E(NR) \geq \underline{b}(y_1, P)$. Under the intuitive criterion, the unique* *separating equilibrium (if it exists) is characterized by*

$$b^E(0; AI, NR) = b^E(0; FI, NR)$$

and

$$b^P(1; AI, NR) \in \arg\max_{b \in A(NR)} U^P(b, q(b, 1))$$

The additional complications in notation arise because the PM's value function $U^\theta(b, q(b, \mu))$ isn't necessarily a single-peaked function of debt, when default is endogenous. For "well behaved" probability distributions F that lead to single-peaked utility functions, the conditions in Lemma 9 reduce to $b^P(1; AI, NR) \in \min\{b^P(1; FI, NR), \hat{b}^E(1, b^E(0; AI, NR))\}$, where $\hat{b}^E(1, b^E(0; AI, NR))$ is defined as in the section with exogenously given default rates.

B.2 Unobservable types - Pooling Equilibrium

Lemma 10 *In a model of asymmetric information with no fiscal rule and endogenous default rates, a pooling equilibrium exists if and only if* $\underline{b}(y_1, P) \geq \hat{b}^E(NR)$.

This means that when $\underline{b}(y_1, P) < \hat{b}^E(NR)$ the separating equilibrium described in lemma 9 is the unique equilibrium.

B.3 Unobservable types - Separating Equilibrium with Debt Limit

Let $A(DL)$ denote the set of debt levels, that are below the full-information debt level chosen by the extravagant PM, and that the extravagant PM wouldn't choose, even if it obtained the borrowing rate of the prudent PM given the debt limit. This is given by

$$A(DL) = \{b < b^E(0; FI, DL) | U^E(b^E(0; FI, DL), q(b^E(0; FI, DL), 0)) \geq U^E(b, q(b, 1))\}.$$

Further, let $\hat{b}^E(DL) = \max_{b \in A(DL)} b$, denote the highest level of debt that is in that set.

Lemma 11 *In a model of asymmetric information with a debt limit and endogenous default rate, a separating equilibrium exists if and only if* $\hat{b}^E(DL) \geq \underline{b}(y_1, P)$. *Under the intuitive criterion, the unique separating equilibrium (if it exists) is characterized by*

$$b^E(0; AI, DL) = b^E(0; FI, DL)$$

and

$$b^P(1; AI, DL) \in \arg\max_{b \in A(DL)} U^P(b, q(b, 1))$$

Since $U^E(b^E(0; FI, DL), q(b^E(0; FI, DL), 0)) \leq U^E(b^E(0; FI, NR), q(b^E(0; FI, NR), 0))$, it follows that $A(DL) \subseteq A(NR)$. In other words, the debt limit shrinks the set of debt levels that allow the prudent PM to signal her type. This implies that

$$U^P(b^P(1; AI, DL), q(b^P(0; AI, DL), 1)) \leq U^P(b^P(0; AI, NR), q(b^P(1; AI, NR), 1))$$

The debt limit makes signalling more difficult for the prudent PM. This must make the prudent government (weakly) worse off with the debt limit. Similarly, citizens are worse off worse off with a debt limit than without, when the prudent government is in power.

B.4 Unobservable types - Pooling Equilibrium with Debt Limit

Lemma 12 *In a model of asymmetric information with debt limit and endogenous default rates, a pooling equilibrium exists if and only if* $\underline{b}(y_1, P) \geq \hat{b}^E(DL)$.

C Proofs

Proof of Observation 1. First notice that $q^E < q^P$, as $\delta^E > \delta^P$. Suppose for contradiction that $b^E(0; FI, NR) \leq b^P(1; FI, NR)$, then

$$
\begin{aligned}
&u'(y_1 + q^E b^E(0; FI, NR)) = \beta^E R u'(y_2 - b^E(0; FI, NR)) \\
<&\beta^P R u'(y_2 - b^P(1; FI, NR)) = u'(y_1 + q^P b^P(1; FI, NR)) < u'(y_1 + q^E b^E(0; FI, NR)),
\end{aligned}
$$

where the equalities follow from the first order conditions. The first inequality follows as $y_2 - b^E(0; FI, NR) \geq y_2 - b^P(1; FI, NR)$, $\beta^E < \beta^P$ and u is concave. The second inequality follows since $q^E < q^P$, $b^E(0; FI, NR) \leq b^P(1; FI, NR)$ and u is concave. Thus, we conclude that $b^E(0; FI, NR) > b^P(1; FI, NR)$ ■

Proof of Observation 2. If the social welfare planner could choose debt freely, it would maximize the following objective function when the PM is of type θ:

$$U^{SWP} = u(y_1 + q^\theta b) + \beta^P[(1 - \delta^\theta)u(y_2 - b) + \delta^\theta u(y_2)],$$

i.e. the social planner takes into account the borrowing rate and default rate of the PM that is in power, but chooses borrowing based on social preferences, reflected in the discount rate β^P.

The optimality condition gives

$$u'(y_1 + q^\theta b^{SWP}(\theta)) = \beta^P R u'(y_2 - b^{SWP}(\theta)), \tag{19}$$

where $b^{SWP}(\theta)$ gives the ideal borrowing when the PM is of type θ. This gives $b^{SWP}(P) = b^P(1, FI, NR)$, because the first order condition is identical to that of the prudent type when $q = q^P$. This also gives $b^{SWP}(E) = b^P(0, FI, NR)$, i.e. the debt that a prudent government would choose if the market perceived it to be extravagant and offered it the bond price q^E.

Optimal borrowing $b^{SWP}(\theta)$ is decreasing in q, so $b^P(1, FI, NR) < b^P(0, FI, NR)$ and the social planner can achieve the first best by imposing a debt limit of $\bar{b} = b^P(0, FI, NR)$, which will bind for the extravagant type, but won't affect the prudent government, who is already borrowing at the socially optimal rate given the bond price they face.

■

Proof of Lemma 1. This proof proceeds in steps. In step 1, we show that $b^P(1; AI, NR) \in A$, where $A = [\hat{b}^P(1, b^P(0; FI, NR)), \hat{b}^E(1, b^E(0; FI, NR))]$ is sufficient for inequality 7 and 8 to hold simultaneously. Step 2 establishes that this is also necessary. Finally, in step 3 we show that under the intuitive criterion any separating equilibrium results in the debt levels are as stated in Lemma 1.

In step 1 and 2, we will assume that the beliefs in the bonds market are given by $\mu(b) = 0$ for every $b \neq b^P(1; AI, NR)$.

Step 1: Consider $b \in A$. Note that $b \leq \hat{b}^E(1, b^E(0; FI, NR)) < b^E(0; FI, NR)$, and $U^E(\cdot, q^E)$ is increasing in the first argument for any $b' < b^E(0; FI, NR)$. Thus, it follows immediately that inequality 7 holds for $b^P(1; AI, NR) = b$. Similarly, $U^P(b^P(0; FI, NR), q^E) \leq U^P(b', q^P)$ for every $b' \geq \hat{b}^P(1, b^P(0; FI, NR))$.

Thus, all that remains to be shown is that $A \neq \emptyset$. Suppose for contradiction that $\hat{b}^P(1, b^P(0; FI, NR)) > \hat{b}^E(1, b^E(0; FI, NR))$. This implies that $U^E(\hat{b}^P(1, b^P(1; FI, NR)), q^P) > U^E(\hat{b}^E(1, b^E(0; FI, NR)), q^P)$. Furthermore, by definition $U^E(b^P(0; FI, NR), q^E) < U^E(b^E(0; FI, NR), q^P)$. Thus, $U^E(\hat{b}^P(1, b^P(0; FI, NR)), q^P) > U^E(b^P(0; FI, NR), q^E)$, which implies

$$
\begin{aligned}
&\beta^E(1-\delta^E)u(y_2 - \hat{b}^P(1, b^P(0; FI, NR))) + \beta^E\delta^E u(y_2) - \beta^P(1-\delta^P)u(y_2 - \hat{b}^P(1, b^P(0; FI, NR))) - \beta^P\delta^P u(y_2) \\
>&\beta^E(1-\delta^E)u(y_2 - b^P(0; FI, NR)) + \beta^E\delta^E u(y_2) - \beta^P(1-\delta^P)u(y_2 - b^P(0; FI, NR)) - \beta^P\delta^P u(y_2) \\
\Rightarrow&\beta^E(1-\delta^E)\left[u(y_2 - \hat{b}^P(1, b^P(0; FI, NR))) - u(y_2 - b^P(0; FI, NR))\right] \\
>&\beta^P(1-\delta^P)\left[u(y_2 - \hat{b}^P(1, b^P(0; FI, NR))) - u(y_2 - b^P(0; FI, NR))\right] \\
>&\beta^E(1-\delta^E)\left[u(y_2 - \hat{b}^P(1, b^P(0; FI, NR))) - u(y_2 - b^P(0; FI, NR))\right],
\end{aligned}
$$

where the first inequality follows from the definition of $\hat{b}^P(1, b^P(0; FI, NR))$, and the third inequality follows since u is increasing, $\hat{b}^P(1, b^P(0; FI, NR)) < b^P(0; FI, NR)$, and $\beta^E(1-\delta^E) < \beta^P(1-\delta^P)$. Thus, we conclude that $b \in A$ is a sufficient condition for a separating equilibrium.

Step 2: To show that there exists no $b \notin A$ that satisfy both inequality 7 and 8, we note that there also exists $\hat{b}^P(1, b^P(0; FI, NR)) > b^P(0; FI, NR)$ and $\hat{b}^E(1, b^E(0; FI, NR)) > b^E(0; FI, NR)$. What remains to be shown is that $\hat{b}^P(1, b^P(0; FI, NR)) < \hat{b}^E(1, b^E(0; FI, NR))$, since

(1) $U^E(b^E(0; FI, NR), q^P) \leq U^E(b, q^P)$ if and only if $b \geq \hat{b}^E(1, b^E(0; FI, NR))$ for every $b > b^E(0; FI, NR)$, and

(2) $U^P(b^P(0; FI, NR), q^E) \geq U^P(b, q^P)$ if and only if $b \leq \hat{b}^P(1, b^P(0; FI, NR))$ for every $b > b^P(0; ; NR)$.

Suppose for contradiction that $\hat{b}^P(1, b^P(0; FI, NR)) \geq \hat{b}^E(1, b^E(0; FI, NR))$. This implies that $U^P(\hat{b}^E(1, b^E(0; FI, NR)), q^P) \geq U^P(b^E(0; FI, NR), q^E)$, and therefore:

$$
\begin{aligned}
&\beta^P(1-\delta^P)u(y_2-\hat{b}^E(1,b^E(0;FI,NR)))+\beta^P\delta^P u(y_2)-\beta^E(1-\delta^E)u(y_2-\hat{b}^E(1,b^E(0;FI,NR)))-\beta^E\delta^E u(y_2)\\
\geq&\beta^P(1-\delta^P)u(y_2-b^E(0;FI,NR))+\beta^P\delta^P u(y_2)-\beta^E(1-\delta^E)u(y_2-b^E(0;FI,NR))-\beta^E\delta^E u(y_2)\\
\Rightarrow&\beta^P(1-\delta^P)\left[u(y_2-\hat{b}^E(1,b^E(0;FI,NR)))-u(y_2-b^E(0;FI,NR))\right]\\
\geq&\beta^E(1-\delta^E)\left[u(y_2-\hat{b}^E(1,b^E(0;FI,NR)))-u(y_2-b^E(0;FI,NR))\right],
\end{aligned}
$$

where the first inequality follows from the definition of $\hat{b}^E(1,b^E(0;FI,NR))$. Since $\hat{b}^E(1,b^E(0;FI,NR)) > b^E(0;FI,NR)$, and $\beta^E(1-\delta^E) < \beta^P)(1-\delta^P)$, this leads to a contradiction.

Step 3: Since $b^E(0;AI,NR)$ is the amount of bond issued by the extravagant government in every separating equilibrium, the intuitive criterion imply that $\mu(b)=1$ for every $b < \hat{b}^E(1,b^E(0;FI,NR)) < b^E(0;FI,NR)$. Thus, the only separating equilibrium (if it exists) that survives the intuitive criterion is the one stated in the Lemma. Furthermore, a separating only exists when $\underline{b}(y_1) \leq \hat{b}^E(1,b^E(0;FI,NR))$. ■

Proof of Lemma 2. Note that in order for b to be the equilibrium bond level issued in a pooling equilibrium the following two inequalities must hold:

$$U^E(b^E(0;FI,NR),q^E) \leq U^E(b,q(\pi)), \tag{20}$$
$$U^P(b^P(0;FI,NR),q^E) \leq U^P(b,q(\pi)). \tag{21}$$

We proceed in steps. In Step 1, we show that a necessary condition for inequalities 20 and 21 to hold simultaneously is for

$$b \in A = [\hat{b}^E(\pi,b^E(0;FI,NR)),\hat{b}^P(\pi,b^P(0;FI,NR))]$$

with $\hat{b}^E(\pi,b^E(0;FI,NR)) < b^E(0;FI,NR)$ and $\hat{b}^P(\pi,b^P(0;FI,NR)) > b^P(0;FI,NR)$. Step 2 establishes that under the intuitive criterion no pooling equilibrium exists if $\underline{b}(y_1) < \hat{b}^E(1,b^E(0;FI,NR))$. In Step 3, we show that if $\underline{b}(y_1) \geq \hat{b}^E(1,b^E(0;FI,NR))$, then a pooling equilibrium exists.

Step 1: If $b \leq \hat{b}^E(\pi,b^E(0;FI,NR))$, then inequality 20 will be violated, and $b \geq \hat{b}^P(\pi,b^P(0;FI,NR))$ would violate inequality 21. Furthermore, if (1) $\hat{b}^E(\pi,b^E(0;FI,NR)) < b^E(0;FI,NR)$ and $\hat{b}^P(\pi,b^P(0;FI,NR)) < b^P(0;FI,NR)$, then by the analogous arguments to Step 1 in the proof of Lemma 1, we have $\hat{b}^E(\pi,b^E(0;FI,NR)) > \hat{b}^P(\pi,b^P(0;FI,NR))$, and (2) $\hat{b}^E(\pi,b^E(0;FI,NR)) > b^E(0;FI,NR)$ and $\hat{b}^P(\pi,b^P(0;FI,NR)) > b^P(0;FI,NR)$, then by the analogous arguments to Step 2 in the proof of Lemma 1, we have $\hat{b}^E(\pi,b^E(0;FI,NR)) > \hat{b}^P(\pi,b^P(0;FI,NR))$.

Step 2: Since inequality 20 holds for any $b \in A$, then the intuitive criterion implies that $\mu(b)=1$

for every $b < \hat{b}^E(1, b^E(0; FI, NR))$. Thus, if $\underline{b}(y_1) < \hat{b}^E(1, b^E(0; FI, NR))$, then the Prudent government would deviate.

Step 3: Suppose that $\underline{b}(y_1) \geq \hat{b}^E(1, b^E(0; FI, NR))$. For $b^{Pool} = \max\{\underline{b}(y_1), \hat{b}^E(\pi, b^E(0; FI, NR))\}$ the intuitive criterion has no bite for feasible bond-levels. Thus,b^{Pool} as the equilibrium bond-level, and beliefs $\mu(b^{Pool}) = \pi$ and $\mu(b) = 0$ for any $b \neq b^{Pool}$ constitute a pooling equilibrium. This completes the proof. ■

Proof of Lemma 3. Whenever, $\bar{b} > b^P(0; FI, NR)$, the proof follows the same arguments as the proof of Lemma 1.

When $\bar{b} \leq b^P(0; FI, NR)$, we have $b^E(0; AI, DL) = b^P(0; FI, DL) = \bar{b}$. Suppose for contradiction that $\hat{b}^E(1, b^E(0; AI, DL)) < \hat{b}^P(1, b^P(0; FI, DL)) \leq \bar{b}$. This implies that $U^E(b^E(0; AI, DL), q^E) = U^E(\hat{b}^E(1, b^E(0; AI, DL)), q^P) < U^E(\hat{b}^P(1, b^P(0; FI, DL)), q^P)$. Thus we have:

$$
\begin{aligned}
&\beta^E(1-\delta^E)u(y_2 - \hat{b}^P(1, b^P(0; FI, DL))) + \beta^E\delta^E u(y_2) - \beta^P(1-\delta^P)u(y_2 - \hat{b}^P(1, b^P(0; FI, DL))) - \beta^P\delta^P u(y_2) \\
>&\beta^E(1-\delta^E)u(y_2 - b^P(0; FI, DL)) + \beta^E\delta^E u(y_2) - \beta^P(1-\delta^P)u(y_2 - b^P(0; FI, DL)) - \beta^P\delta^P u(y_2) \\
\Rightarrow&\beta^E(1-\delta^E)\left[u(y_2 - \hat{b}^P(1, b^P(0; FI, DL))) - u(y_2 - b^P(0; FI, DL))\right] \\
>&\beta^P(1-\delta^P)\left[u(y_2 - \hat{b}^P(1, b^P(0; FI, DL))) - u(y_2 - b^P(0; FI, DL))\right] \\
>&\beta^E(1-\delta^E)\left[u(y_2 - \hat{b}^P(1, b^P(0; FI, DL))) - u(y_2 - b^P(0; FI, DL))\right],
\end{aligned}
$$

where the first inequality follows from the definition of $\hat{b}^P(1, b^P(0; FI, DL))$, and the third inequality follows since u is increasing and $\hat{b}^P(1, b^P(0; FI, DL)) < b^P(0; FI, DL)$, and $\beta^E(1-\delta^E) < \beta^P(1-\delta^P)$. Thus, we have a contradiction.

The intuitive equilibrium again imply that the unique separating equilibrium is one in which $b^E(0; AI, DL) = \bar{b}$, and $b^P(1; AI, DL) = \hat{b}^E(1, b^E(0; AI, DL))$ ■

Proof of Lemma 4. This proof is analogous to the proof of Lemma 2. ■

Proof of Observation 3. First we observe that $b^P(0; FI, NR) < b^E(0; FI, NR)$. This follows directly from the first order conditions, and $\beta^E < \beta^P$.

Suppose that the socially optimal level of debt limit is $\bar{b} \geq b^E(0; FI, NR)$. Consider a decrease of the borrowing limit to $\bar{b}' = b^E(0; FI, NR) - \varepsilon$, where $\varepsilon > 0$ is small. Since $b^E(0; FI.NR)$ is the first best for the extravagant government, we have

$$
U^E(b^E(0; FI, NR), q^E) \simeq U^E(\bar{b}', q^E) - \varepsilon U^E_2(b^E(0; FI, NR), q^E),
$$

where the second term on the RHS is zero by the envelope theorem. Thus, the decrease to the borrowing limit has no effect on the utility of the extravagant government. By extension this has no effect on the equilibrium level of bonds issued by the prudent government compared to the case without a binding debt limit. The decrease in the bond-level issued by the extravagant government increases the social welfare, as $b^S < b^E(0; FI, NR)$.

Similarly, if $\bar{b} = b^S$, then the social cost of increasing the borrowing limit slightly will not impact the social welfare when the extravagant government is in power. However it will increase the utility of the extravagant government as $b^E(0; FI, NR) > b^S$. This in turn relaxes the constraint for the prudent government. As $b^P(1; FI, NR) < b^S$ this increases the social welfare. ■

Proof of Lemma 5. Given debt b and second period revenue y_2, default is optimal for a type θ government if and only if

$$u((1-\gamma)y_2) + \beta^\theta u((1-\gamma)y_3) \geq u(y_2 - b) + \beta^\theta u(y_3).$$

Since the RHS is decreasing in b this proves the claim. ■

Proof of Lemma 6. Given debt b and second period revenue y_2, default is optimal for a type θ government if and only if

$$u((1-\gamma)y_2) - u(y_2 - b) \geq \beta^\theta \left(u(y_3) - u((1-\gamma)y_3)\right) > 0,$$

where the second inequality follows since $\gamma \in (0,1)$. Therefore a necessary condition for a type θ government to default is $(1-\gamma)y_2 > y_2 - b$. Since u is increasing and concave if follows that if the above inequality holds for some y_2, then it holds for all $y_2' \leq y_2$. ■

Proof of Lemma 7. This follows directly from definition 3, lemma 6, and $\beta^P > \beta^E$. ■

Proof of Lemma 8. Consider a bond price schedule $q(b)$ and $b > b'$. The difference in utility as $b' \lim b$:

$$\lim_{b' \to b} (u(y_1 + q(b)b) - u(y_1 + q(b')b')) - \beta \int_{y(b,\beta)}^{\bar{y}} u'(y_2 - b)F(y)$$
$$- f(y(b,\beta))\frac{\partial y(b,\beta)}{\partial b}[u(y(b,\beta) - b) + \beta u(y_3) - u((1-\gamma)y(b,\beta)) - \beta u((1-\gamma)y_3)].$$

Where the first term does not depend on β, and the third term is zero given the optimal choice of default in period 2.

By lemma 7 $y(b,\beta)$ is decreasing in β, thus $\int_{y(b,\beta)}^{\bar{y}} u'(y_2 - b)F(y)$ is increasing in β. Hence $\beta \int_{y(b,\beta)}^{\bar{y}} u'(y_2 - b)F(y)$ is increasing in β which completes the proof. ■

Proof of lemma 9. In order for $b^P(0; AI, NR)$ and $b^E(0; AI, NR)$ to be the equilibrium debt levels choosen by the prudent and extravagant government, respectively, then the following two inequalities need to hold simulatenously:

$$U^E(b^E(0;AI,NR),q(b^E(0;AI,NR),0)-U^E(b^P(1;AI,NR),q(b^P(1;AI,NR),1)\geq 0 \tag{22}$$
$$U^P(b^P(1;AI,NR),q(b^P(1;AI,NR),1)-U^P(b^P(0;FI,NR),q(b^P(0;FI,NR),0)\geq 0 \tag{23}$$

where the first inequality ensures that the extravagant PM does not want to mimic the prudent PM, and the second inequality ensures that the prudent PM does not prefer to be mistaken for an extravagant PM.

To show existence of a separating equilibrium we therefore need to show that there exists a b such that the inequalities above hold. To this end, consider the net benefit for each PM type of getting $b = b^P(0;FI,NR)$ and being perceived as the prudent PM, and their their optimal choice when being perceived as the extravagant PM. Let $b^E = b^E(0;AI,NR)$, then

$$U^P(b,q(b,1))-U^P(b,q(b,0))=u(y_1+q(b,1)b)-u(y_1+q(b,0)b)=K>0$$

where the inequality follows from Lemma 7, and the fact that default choices in period 2 do not depend on the bond prices at which the debt level was acquired in period 1. For the extravagant this is:

$$U^E(b,q(b,1))-U^E(b^E,q(b^E,0))=K+U^E(b,q(b,0))-U^E(b^E,q(b^E,0))\leq K,$$

by optimality of b^E. By Lemma 8, it follows that there exists a b such that the two above inequalities hold simultaneously.

Next we show that there exists no separating equilibrium in which $b^P(1;AI,NR) > b^E(0;FI,NR)$. To this end, consider the benefit for the extravagant type of being perceived as the prudent PM, rather than as the extravagant PM, at the debt level $b = b^E(0;FI,NR)$:

$$U^E(b,q(b,1))-U^E(b,q(b,0))=u(y_1+q(b,1)b)-u(y_1+q(b,0)b)=K>0.$$

where the inequality follows from Lemma 7, and the fact that the default choices in period 2 do not depend on the bond prices at which the debt level was acquired in period 1.

The difference between the prudent PM's utility from requesting debt level b and being perceived as the prudent PM and her optimal debt level when being perceived as the extravagant PM,

$b^P = b^P(0; FI, NR)$, is:

$$U^P(b, q(b,1)) - U^P(b^p, q(b^P, 0)) = K + U^P(b, q(b,0)) - U^P(b^p, q(b^P, 0)) \leq K$$

where the inequality follows from optimality of b^P. By Lemma 8, it follows, that for $b > b^E(0; FI.NR)$ such that the extravagant PM does not want mimic the prudent PM, then the prudent PM prefers to request debt level $b^P(0; FI, NR)$ and being perceived as the extravagant PM. We conclude that no separating equilibrium with upward deviation exists.

Finally, by the intuitive criterion $\mu(b) = 1$ for every $b \in A(NR)$. Hence the unique separating equilibrium that satisfy the intuitive criterion has $b^E(0; AI, NR) = b^E(0; FI, NR)$, and $b^P(1; AI, NR) \in \arg\max_{b \in A(NR)} U^P(b, q(b,1))$. ■

Proof of Lemma 10. For a pooling equilibrium at a debt level b to exist, it has to be the case that

$$U^E(b, q(b,\pi)) \geq U^E(b^E, q(b^E, 0)) \tag{24}$$

$$U^P(b, q(b,\pi)) \geq U^P(b^P, q(b^P, 0)) \tag{25}$$

where $b^\theta = b^\theta(0; FI, NR)$ for $\theta \in \{E, P\}$.

Let $A^E(NR) = \{b : U^E(b, q(b,\pi)) \geq U^E(b^E, q(b^E, 0))\}$, and $A^P(NR) = \{b : U^P(b, q(b,\pi)) \geq U^P(b^P, q(b^P, 0))\}$ denote the values of b for which each of the two constraints hold. Clearly, a necessary condition for a debt level b to be the equilibrium debt level requested by both types of PM's is $b \in A^E(NR) \cap A^P(NR)$. Notice that $U^E(b, q(b,\pi))$ is continuous in b, since F has no atoms. This, combined with Lemma 8, implies that $U^E(b^*, q(b^*,\pi)) = U^E(b^E, q(b^E, 0))$ for $b^* = \min_{b \in A^E(NR) \cap A^P(NR)} b$. Since $U^E(b, q(b,\pi)) \geq U^E(b^E, q(b^E, 0))$ for every $b \in A^E(NR) \cap A^P(NR)$, this implies that for every b' such that $U^E(b, q(b,1)) \leq U^E(b^*, q(b^*,\pi)) = U^E(b^E, q(b^E, 0))$ we have $U^E(b, q(b,1)) \leq U^E(b', q(b',\pi))$ for every $b' \in A^E(NR) \cap A^P(NR)$. By the intuitive criterion, $\mu(b) = 1$ for every $b \in A(NR)$. Thus, if $\max_{b \in A(NR)} b \geq \underline{b}(y_1, q^P)$, then no pooling equilibrium exists that satisfies the intuitive criterion. On the other hand, if $b^E(NR) < \underline{b}(y_1, P)$, then the intuitive criterion is no longer relevant, and the debt level requested from both types of PM's b^* combined with the beliefs $\mu(b^*) = \pi$ and $\mu(b) = 0$ for every $b \neq b^*$ constitutes an equilibrium. ■

Proof of Lemma 11. For $b^P(0; AI, DL)$ and $b^E(0; AI, DL)$ to be the equilibrium debt levels choosen by the prudent and extravagant government, respectively, then the following two inequalities need to hold simulatenously:

$$U^E(b^E(0; AI, DL), q(b^E(0; AI, DL), 0) - U^E(b^P(1; AI, DL), q(b^P(1; AI, DL), 1) \geq 0 \tag{26}$$

$$U^P(b^P(1; AI, NR), q(b^P(1; AI, DL), 1) - U^P(b^P(0; FI, DL), q(b^P(0; FI, DL), 0) \geq 0 \tag{27}$$

where the first inequality ensures that the extravagant PM does not want to mimic the prudent PM, and the second inequality ensures that the prudent PM does not prefer to be mistaken for an extravagant PM.

To show existence of a separating equilibrium we therefore need to show that there exists a b such that the inequalities above hold. To this end, consider the net benefit to each of the PMs of borrowing $b = b^P(0; FI, DL)$ and being perceived as the prudent PM and their optimal choice when being perceived as the extravagant PM. Let $b^E = b^E(0; FI, NR)$, then:

$$U^P(b, q(b,1)) - U^P(b, q(b,0)) = u(y_1 + q(b,1)b) - u(y_1 + q(b,0)b) = K > 0$$

where the inequality follows from Lemma 7, and that the default choices in period 2 do not depend on the bond prices at which the debt level was acquired in period 1. For the extravagant type this is:

$$U^E(b, q(b,1)) - U^E(b^E, q(b^E,0)) = K + U^E(b, q(b,0)) - U^E(b^E, q(b^E,0)) \leq K,$$

by the optimality of b^E, when constrained by the debt limit. If follows from Lemma 8 that there exists a b such that the two above inequalities hold simultaneously.

Next we show that there exists no separating equilibrium in which $b^P(1; AI, DL) > b^E(0; FI, DL)$. To this end consider the difference in utility for the extravagant PM between perceived as the prudent PM and as the extravagant PM, when borrowing $b = b^E(0; FI, DL)$:

$$U^E(b, q(b,1)) - U^E(b, q(b,0)) = u(y_1 + q(b,1)b) - u(y_1 + q(b,0)b) = K > 0.$$

The inequality follows from Lemma 7 and the fact that the default choices in period 2 do not depend on the bond prices at which the debt level was acquired in period 1.

The difference between the prudent PM's utility from requesting debt level b and being perceived as the prudent PM and their optimal debt level when being perceived as the extravagant PM, $b^P = b^P(0; FI, DL)$, is:

$$U^P(b, q(b,1)) - U^P(b^p, q(b^P,0)) = K + U^P(b, q(b,0)) - U^P(b^p, q(b^P,0)) \leq K$$

where the inequality follows from optimality of b^P. By Lemma 8, it follows, that for $b > b^E(0; FI, DL)$ such that the extravagant PM does not want mimic the prudent PM, then the prudent PM prefers to request debt level $b^P(0; FI, DL)$ and being perceived as the extravagant PM. We conclude that no separating equilibrium with upward deviation exists.

Finally, by the intuitive criterion, $\mu(b) = 1$, for every $b \in A(DL)$. Hence the unique separating equilibrium that satisfies the intuitive criterion has $b^E(0; AI, DL) = b^E(0; FI, DL)$, and $b^P(1; AI, DL) \in \arg\max_{b \in A(DL)} U^P(b, q(b, 1))$. ■

Proof of Lemma 12. For a pooling equilibrium with b to exists it has to be the case that

$$U^E(b, q(b, \pi)) \geq U^E(b^E, q(b^E, 0)) \tag{28}$$

$$U^P(b, q(b, \pi)) \geq U^P(b^P, q(b^P, 0)) \tag{29}$$

where $b^\theta = b^\theta(0; FI, DL)$ for $\theta \in \{E, P\}$.

Let $A^E(DL) = \{b : U^E(b, q(b, \pi)) \geq U^E(b^E, q(b^E, 0))\}$, and $A^P(DL) = \{b : U^P(b, q(b, \pi)) \geq U^P(b^P, q(b^P, 0))\}$ denote the values of b for which each of the two constraints hold. Clearly a necessary condition for a debt level b to be the equilibrium debt level requested by both types of PM's is $b \in A^E(DL) \cap A^P(DL)$. Notice that $U^E(b, q(b, \pi))$ is continuous in b, since F has no atoms. This combined with Lemma 8 implies that $U^E(b^*, q(b^*, \pi)) = U^E(b^E, q(b^E, 0))$ for $b^* = \min_{b \in A^E(DL) \cap A^P(DL)} b$. Since $U^E(b, q(b, \pi)) \geq U^E(b^E, q(b^E, 0))$ for every $b \in A^E(DL) \cap A^P(DL)$, this implies that for every b' such that $U^E(b, q(b, 1)) \leq U^E(b^*, q(b^*, \pi)) = U^E(b^E, q(b^E, 0))$ we have $U^E(b, q(b, 1)) \leq U^E(b', q(b', \pi))$ for every $b' \in A^E(DL) \cap A^P(DL)$. By the intuitive criterion, $\mu(b) = 1$ for every $b \in A(DL)$. Thus, if $\max_{b \in A(DL)} b \geq \underline{b}(y_1, q^P)$, then no pooling equilibrium exists under the intuitive criterion. On the other hand, if $b^E(DL) < \underline{b}(y_1, P)$, then the intuitive criterion has on bite, and the debt level request from both types of PM's b^* combined with the beliefs $\mu(b^*) = \pi$ and $\mu(b) = 0$ for every $b \neq b^*$ constitutes an equilibrium. ■